14th APCTP-BLTP JINR Joint Workshop
- Memorial Workshop in Honor of Prof. Yongseok Oh

Parton distribution functions of the nucleon in the large N_c limit

In collaboration with
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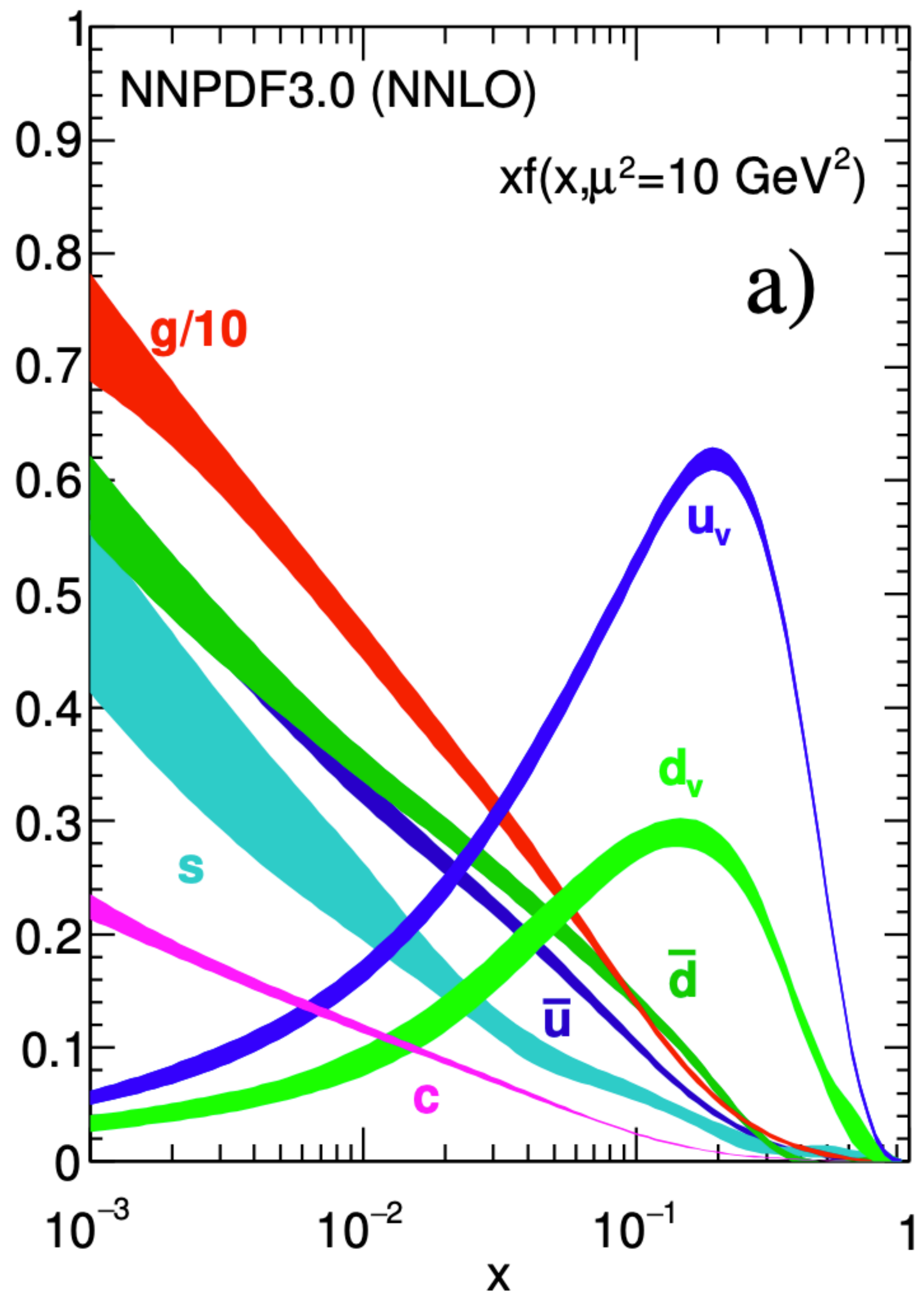
Introduction

Parton distribution functions (PDFs)

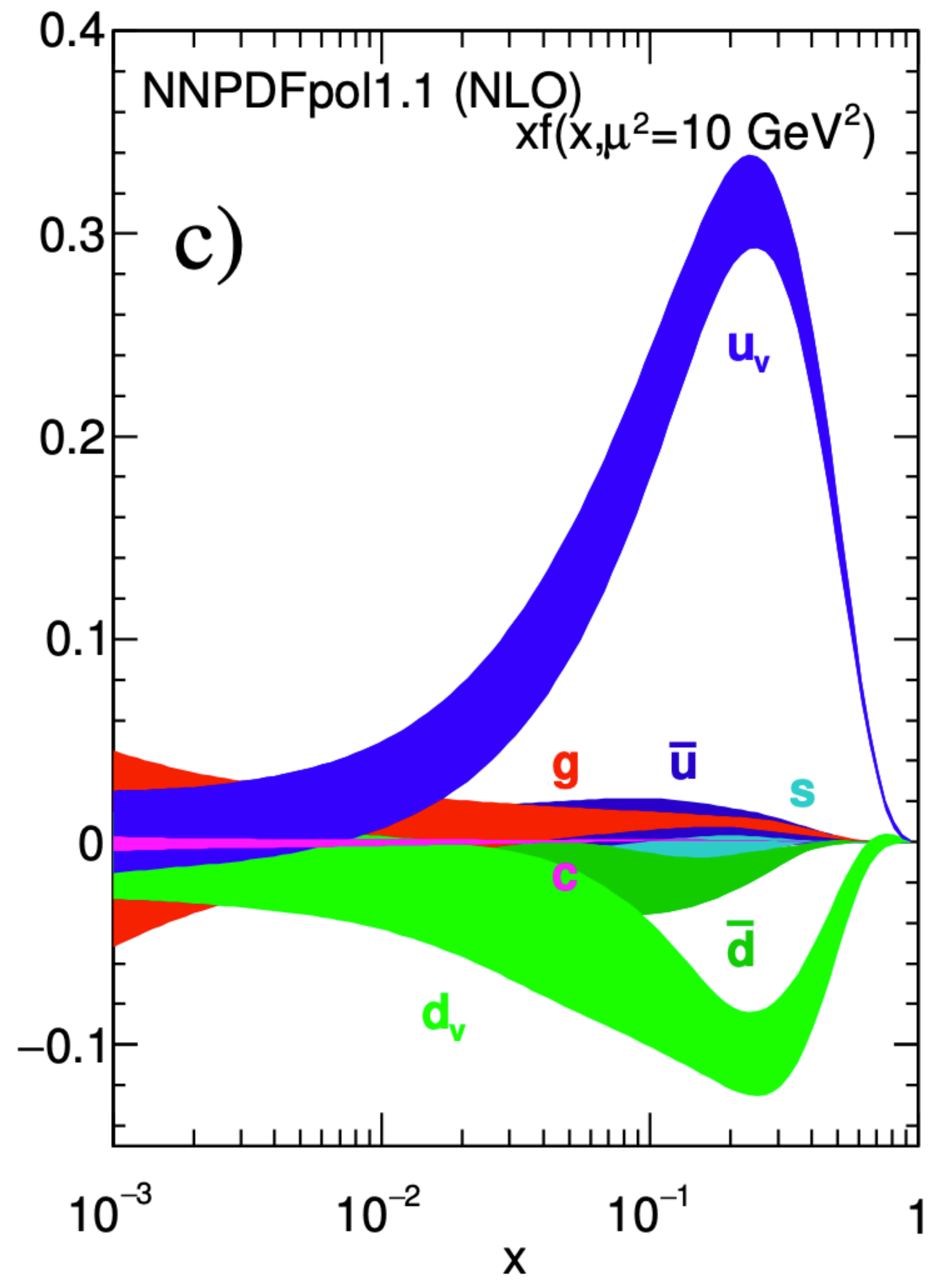
How partons (quarks and gluons) are distributed inside a hadron

Probability density (properly defined on the light-cone)

Proton, global analyses, plots from PDG 2021



R. D. Ball et al. (NNPDF), JHEP 04, 040 (2015)



E. R. Nocera et al. (NNPDF), Nucl. Phys. B887, 276 (2014)

Parton distribution functions (PDFs)

Universality

PDFs do not distinguish different types of reactions

eg. Drell-Yan process (pp collision)

Fitting model PDFs using various reactions (Global analysis)

Justification of factorisation is essential but mostly assumed

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution (1970')

Perturbative evolution of PDFs

$$\frac{dq_i(x,\mu^2)}{\partial\mu^2} = P_{qq}\otimes q_i + P_{qg}\otimes g$$

Splitting functions P_{ij}: probability of perturbative emission of i from j

Twist-2 quark distribution functions

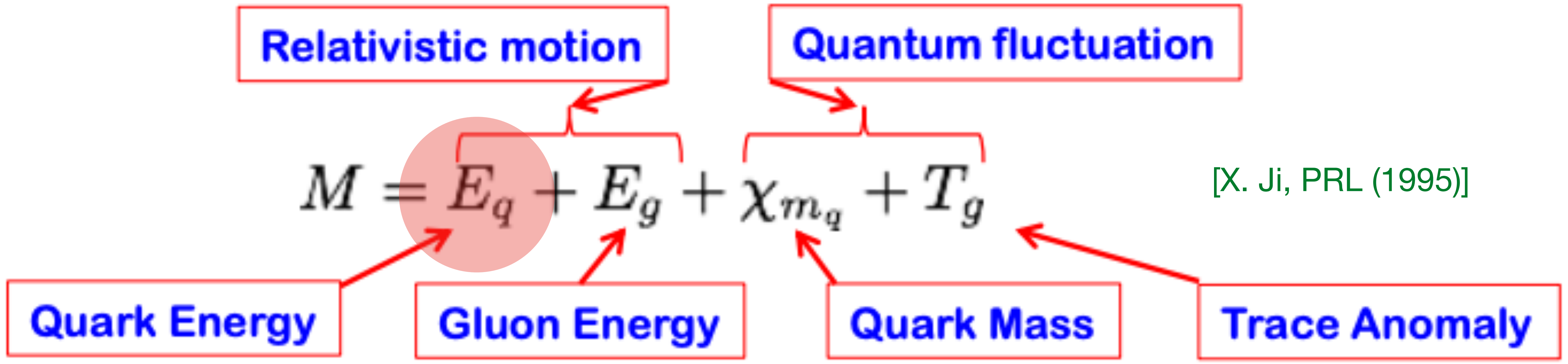
Unpolarized quark distributions

Probability to find a quark with momentum fraction x ~ dx

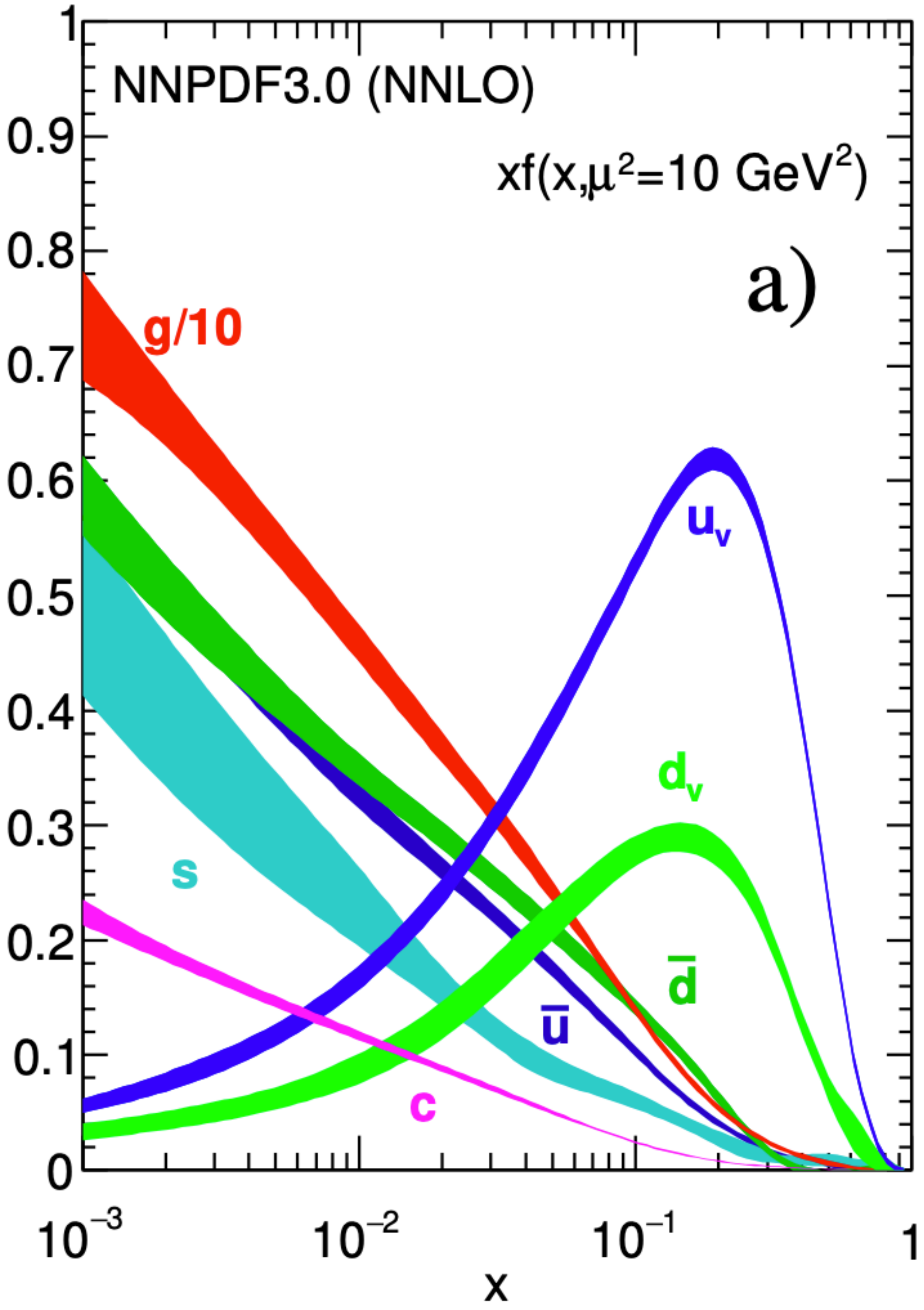
Baryon number and momentum sum rules

→ Momentum sum-rule: Mass form factor (EMT)

→ Mass decomposition



[X. Ji, PRL (1995)]

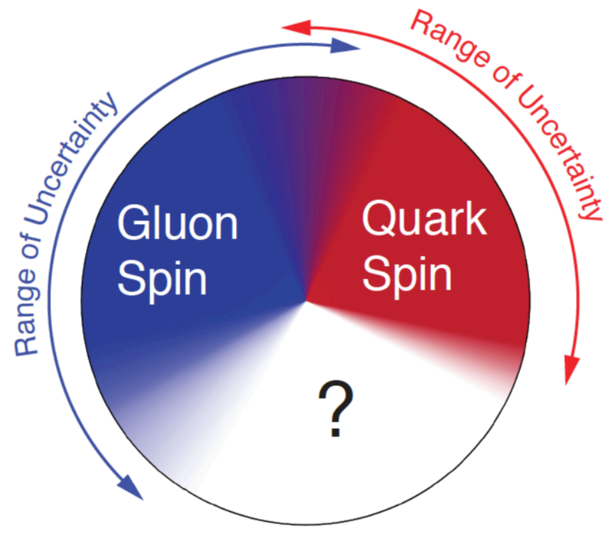


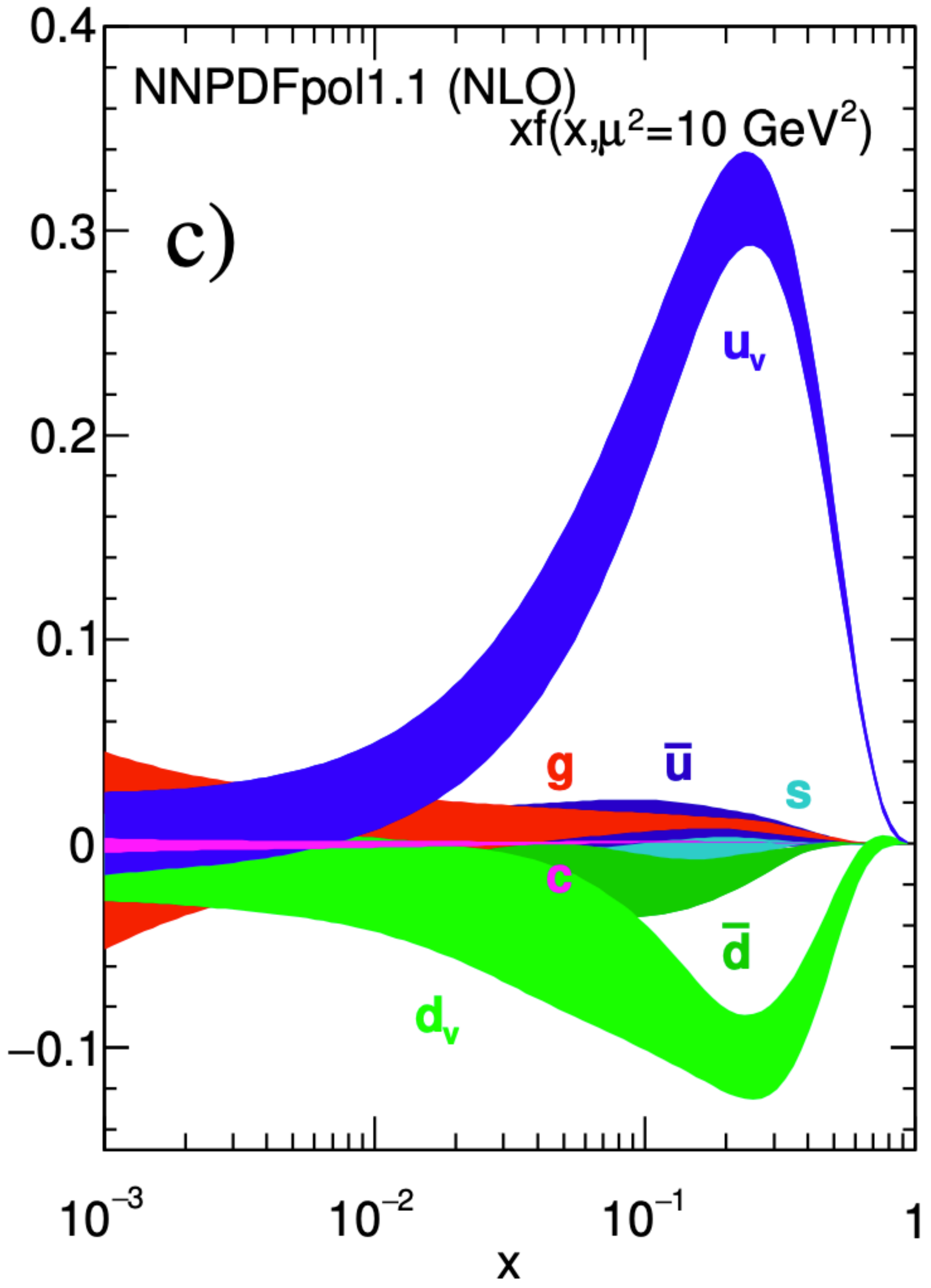
Twist-2 quark distribution functions

Longitudinally polarized quark distribution

Spin sum-rule and axial charge

→ Proton spin decomposition





$$1/2 = \frac{1}{2}\int_0^1 dx\ \Delta\Sigma(x,Q^2) + \int_0^1 dx\ \Delta g(x,Q^2) + \sum_q L_q + L_g$$

[Jaffee, Manohar, NPB 337 (1990)]

Theoretical understanding of PDFs

PDFs are non perturbative objects

Effective models (at low renormalization scale)

- provide initial conditions of the QCD evolution (quark distributions)
- Necessary conditions: Positivity, sum-rules (number, momentum, Bjorken)
- Predictions:
  understanding the non-perturbative phenomena in terms of the effective degrees of freedom

Lattice QCD

- fundamental difficulties being Euclidean: no direct computation is possible
- Mostly studied using the Mellin moments of the PDFs (large noise at higher moments)

Quasi parton distribution function

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

$$q(x,\mu,P^z) = \int \frac{dz}{4\pi} e^{-ixP^z z} \langle P|\bar{\psi}(0)\gamma^z \exp\left[-ig\int_0^z dz' A^z(z')\right]\psi(z)|P\rangle + \mathcal{O}(\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}, \frac{M_N^2}{(P^z)^2})$$

$x \in (-\infty, +\infty)$

μ: renormalization scale
P_z: nucleon momentum

Large **M**omentum **E**ffective **T**heory

Spacelike matrix element → can be calculated on the Lattice

No unique definition → **$\Gamma=\gamma^3$** or **$\Gamma=\gamma^0$**

Approaches the PDFs in the limit Pz →∞, or v → 1.

Quasi parton distribution function

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

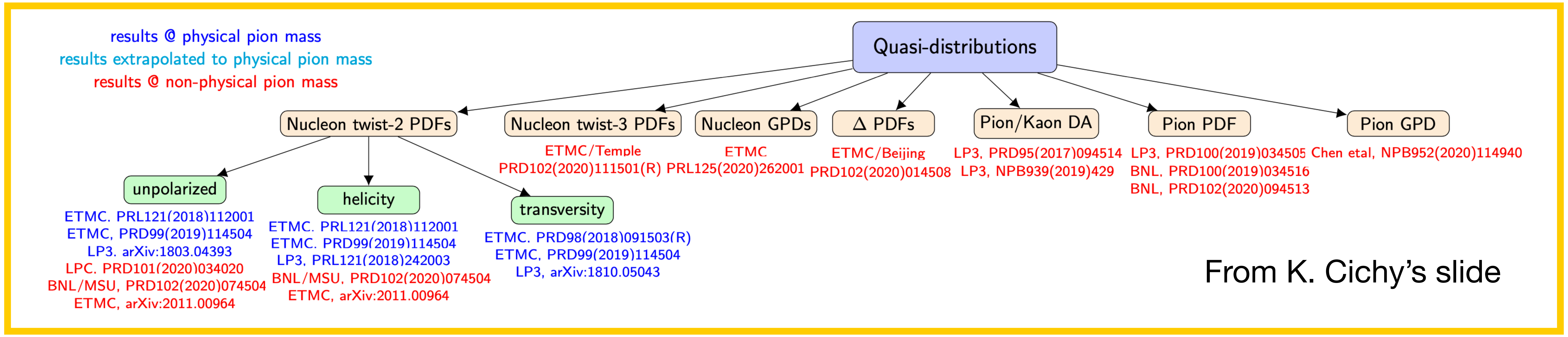
$$q(x,\mu_R,P^z)=\int_{-1}^{1}\frac{dy}{|y|}\underline{C(\frac{x}{y},\frac{\mu_R}{\mu},\frac{\mu}{p^z})}q(y,\mu)+\mathcal{O}(\frac{\Lambda^2_{\rm QCD}}{(P^z)^2},\frac{M_N^2}{(P^z)^2})$$

Perturbative matching coefficients

Extensively studied for the Lattice calculation

Market results P_z ~ 2-3 GeV

N, π, K / PDFs, DAs, GPDs



From K. Cichy's slide

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Enough accuracy and uncertainty for actual application?

Reliable model computations on quasi-PDFs is needed

Review: K. Cichy and M. Constantinou, Adv.High Energy Phys. 2019 (2019) 3036904
Community report: M. Constantinou et al, Prog.Part.Nucl.Phys. 121 (2021) 103908
and many more..

(Quasi-)PDFs in the chiral quark-soliton model

Twist-2 PDFs in a large N_c effective model

[D. Diakonov, V. Y. Petrov, P. V. Pobylitsa, M. Polyakov, and C. Weiss, Nuclear Physics B 480, 341 (1996)]

Initial value at a low renormalization scale μ=1/ρ=600 MeV

Quark and antiquarks: sum-rules, positivity, …

Properties of qPDFs for quarks and antiquarks in the nucleon:

Sum-rules, positivity, evolution in P_z

Gravitational form factor $\bar{c}^q$ is related to the momentum sum-rule:

$\bar{c}^q$ ~ non-convergence of the separate quark EMT operator

Mass decomposition of the nucleon

Interaction strength between the quark- and gluon- subsystems

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Nucleon matrix element in Euclidean separation

Lorentz boost → PDFs ~ quasi-PDF

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Interaction strength between the quark- and gluon- subsystems

[M. Polyakov and HDS, JHEP 09 (2018) 156]

Outline

Chiral quark-soliton model

(quasi-)PDFs in the large Nc

Sum-rules for the quasi-PDFs as their Mellin moments

Numerical results for the isoscalar unpolarized and
isovector longitudinally polarized quark quasi-distributions

Antiquark asymmetries in the proton

Chiral quark-soliton model

Effective partition function from the instanton vacuum

[D. Diakonov, V. Petrov, and P. Pobylitsa, Nucl. Phys. B 306, 809 (1988)]

$$Z = \int \mathcal{D}\pi^a d\psi^\dagger d\psi \ \exp \int d^4x \psi^\dagger(x)(i\partial\!\!\!/ + iMU^{\gamma_5})\psi(x)$$

$$U^{\gamma_5}(x) = U(x)\frac{1+\gamma_5}{2} + U^\dagger(x)\frac{1-\gamma_5}{2} \qquad U(x) = \exp\left[\frac{i}{F_\pi}\pi^a(x)\tau^a\right]$$

Low energy effective theory derived from QCD via the **instantons**

Instanton parameters: average size $\bar{\rho} \sim 1/3 \text{ fm}$ & distance $\bar{R} \sim 1 \text{ fm}$ (no more parameters, Λ_{QCD})

Intrinsic renormalisation scale $\Lambda \sim 1/\bar{\rho} \approx 600 \text{ MeV}$

Spontaneous chiral symmetry breaking & dynamically generated quark mass $M = 350 \text{ MeV}$

Fully field theoretic: successfully describes a wide class of baryon properties

Nucleon: chiral soliton in the large Nc, quarks are bound by a self-consistent mean-field

Interplays the quark-model and (topological) soliton picture of the baryons

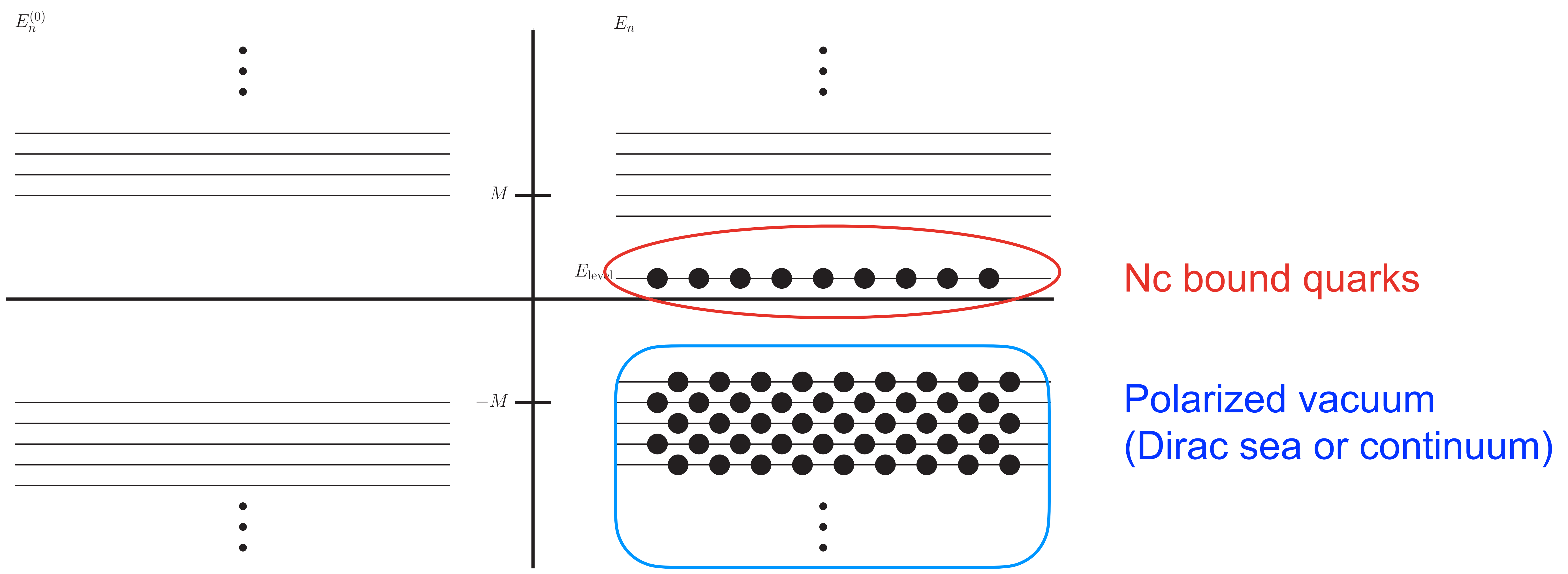
[E. Witten, Nucl. Phys. B 160, 57 (1979)]

Nucleon as a chiral soliton in the large N_c limit

Nc quarks are bound by a

pion mean-field, self-consistently

generated by their interactions



Hedgehog Ansatz

$$U = \exp[i\gamma_5 \hat{n}^a \tau^a P(r)]$$

Dirac spectra (n): Grandspin K= J + T and Parity P

$$H\Phi_n(\vec{x}) = E_n \Phi_n(\vec{x})$$

Classical soliton energy

$$\frac{\delta}{\delta U}(N_c E_{\rm level} + E_{\rm cont.})|_{U=U_c} = 0 \quad \rightarrow \quad M_{sol} = N_c E_{\rm level}(U_c) + E_{\rm cont.}(U_c)$$

Nucleon quantum numbers: quantization around the rotational zero-modes

quasi-PDFs in the χQSM

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808 (2020) 135665]

[HDS, Phys.Lett.B 838 (2023) 137741]

Twist-2 quark distribution functions

In general, in the large Nc limit:

Isosinglet unpolarised	$u(x) + d(x)$	$\sim N_c^2\ \rho(N_c x)$
Isovector polarised	$\Delta u(x) - \Delta d(x)$	

Isovector unpolarised	$u(x) - d(x)$	$\sim N_c\ \rho(N_c x)$
Isosinglet polarised	$\Delta u(x) + \Delta d(x)$	

quasi-PDFs acquire overall factor of v

→ follow the same Nc ordering

Quasi-PDFs in the χQSM

Nucleon at rest → Lorentz boost to a inertial frame with velocity v in the z direction

Quasi- quark and antiquark number densities

$$D_f(x,v) = \frac{1}{2E_N}\int \frac{d^3k}{(2\pi)^3}\delta\left(x-\frac{k^3}{P_N}\right)\int d^3x\; e^{-i\mathbf{k}\cdot\mathbf{x}}\langle N_v|\bar{\psi}_f\left(-\frac{\mathbf{x}}{2},t\right)\Gamma\psi_f\left(\frac{\mathbf{x}}{2},t\right)|N_v\rangle$$

$$\bar{D}_f(x,v) = \frac{1}{2E_N}\int \frac{d^3k}{(2\pi)^3}\delta\left(x-\frac{k^3}{P_N}\right)\int d^3x\; e^{-i\mathbf{k}\cdot\mathbf{x}}\langle N_v|\mathrm{Tr}\left[\Gamma\psi_f\left(-\frac{\mathbf{x}}{2},t\right)\bar{\psi}_f\left(\frac{\mathbf{x}}{2},t\right)\right]|N_v\rangle$$

become exact number densities in the limit $v \to 1$

Isoscalar unpolarized $x \in (-\infty,\infty)$ $H\Phi_n(\vec{x}) = E_n\Phi_n(\vec{x})$

$$\sum_f q_f(x,v) = N_c M_N v \sum_{n,occ}\int \frac{d^3k}{(2\pi)^3}\delta(k^3+vE_n-vM_Nx)\left[\Phi_n^\dagger(\vec{k})(1+v\gamma^0\gamma^3)\gamma_0\Gamma\Phi_n(\vec{k})\right]$$

$\sim N_c^2$

Isovector polarized (helicity)

$$\Delta u(x,v)-\Delta d(x,v) = -\frac{1}{3}(2T^3)\frac{N_cM_Nv}{2\pi}\sum_{n,occ}\int\frac{d^2k_\perp}{(2\pi)^2}\delta(k^3+vE_n-vM_Nx)$$
$$\left[\Phi_n^\dagger(\vec{k})(1+v\gamma^0\gamma^3)\gamma_0\Gamma\tau^3\gamma^5\Phi_n(\vec{k})\right]$$

$\Gamma=\gamma^0$ and $\Gamma=\gamma^3$ define different quasi-PDFs

Quasi-PDFs in the χQSM

Nucleon at rest $\longrightarrow$ Lorentz boost to a inertial frame with velocity v in the z direction

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become exact number densities in the limit $v \longrightarrow \infty$

Isoscalar unpolarized $x \in (-\infty, \infty)$ $H\Phi_n(\vec{x}) = E_n\Phi_n(\vec{x})$

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$\Gamma = \gamma^0$ and $\Gamma = \gamma^3$ define different quasi-PDFs

Sum-rules

Baryon number

$$\int_{-\infty}^{\infty} dx\; q(x,v) = \begin{cases} N_c B, & \Gamma = \gamma^0 \\ v N_c B, & \Gamma = \gamma^3 \end{cases}$$

Momentum

$$\int_{-\infty}^{\infty} dx\; xq(x,v) = \begin{cases} 1, & \Gamma = \gamma^0 \\ v, & \Gamma = \gamma^3 \end{cases}$$

Bjorken

$$\int_{-\infty}^{\infty} dx\; (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

→ better Dirac matrix Γ for the convergence to the PDFs?

→ Interpretation of the QCD symmetry currents

Sum-rules

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Momentum

$$\int_{-\infty}^{\infty} dx\, x q(x,v) = \begin{cases} 1, & \Gamma = \gamma^0 \\ v, & \Gamma = \gamma^3 \end{cases}$$

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$$\int_{-\infty}^{\infty} dx\, (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

U(1): charge density (γ^0) vs flux (γ^3)

Sum-rules

Baryon number

$$\int_{-\infty}^{\infty} dx\, q(x,v) = \begin{cases} N_c B, & \Gamma = \gamma^0 \\ v N_c B, & \Gamma = \gamma^3 \end{cases}$$

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Momentum sum-rule is satisfied only by quarks

Energy-momentum tensor: momentum flux ($T^{30} \sim \partial_3 \gamma^0$) vs pressure ($T^{33} \sim \partial_3 \gamma^3$)

In general, $M_2^q(\Gamma = \gamma^3) = v\left(A^q(0) - \frac{1-v^2}{v^2}\bar{c}^q(0)\right)$

[Maxim Polyakov and HDS, JHEP 09 (2018) 156]

Sum-rules

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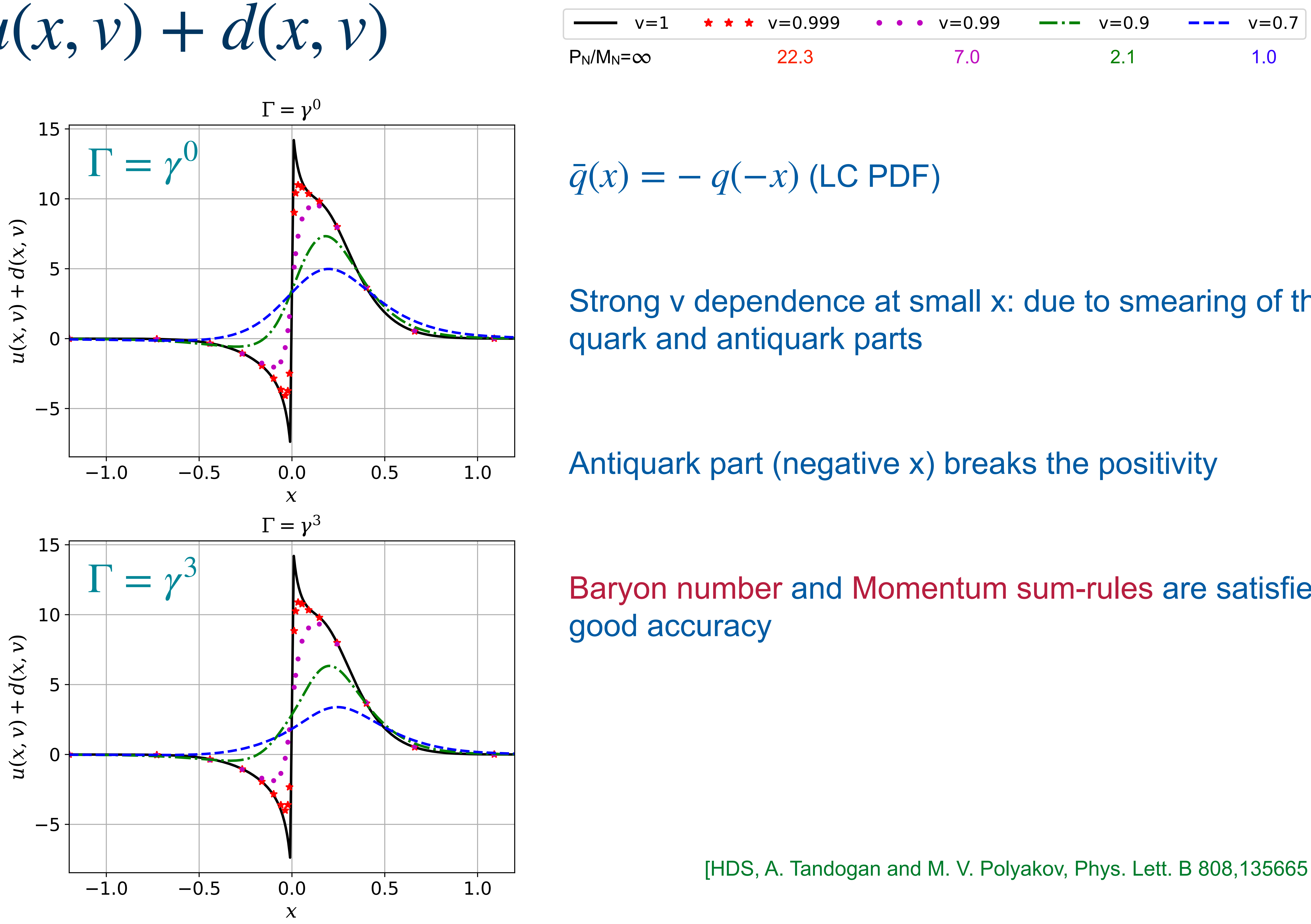
Bjorken

$$\int_{-\infty}^{\infty} dx\ (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

Axial current: $\gamma^3 \sim S^3 g_A^{(3)}$ vs $\gamma^0 \sim \vec{S} \cdot \vec{v} g_A^{(3)}$

$u(x,v) + d(x,v)$



$\bar{q}(x) = -q(-x)$ (LC PDF)

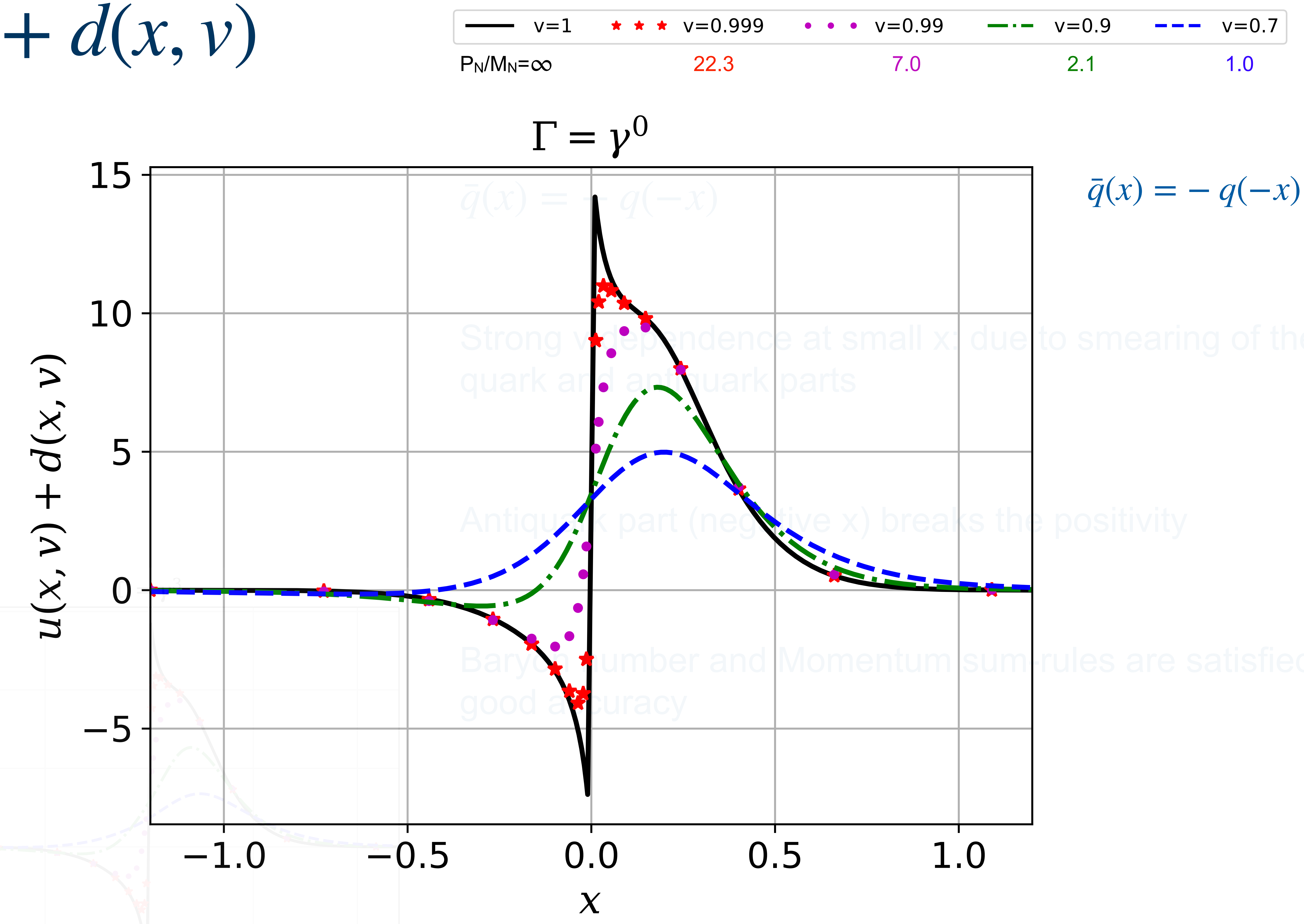
Strong v dependence at small x: due to smearing of the quark and antiquark parts

Antiquark part (negative x) breaks the positivity

Baryon number and Momentum sum-rules are satisfied in good accuracy

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808,135665 (2020)]

$u(x, v) + d(x, v)$



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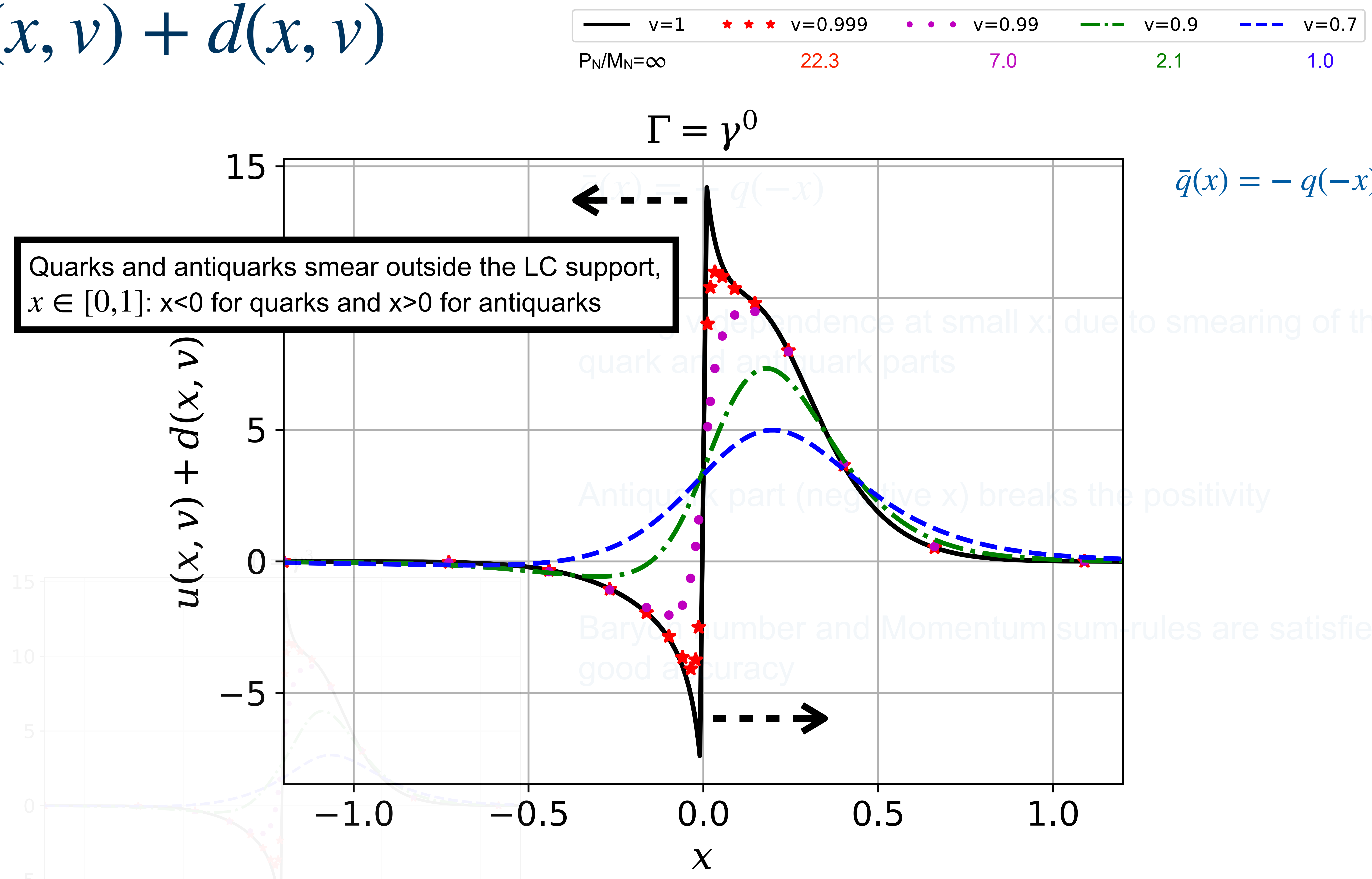

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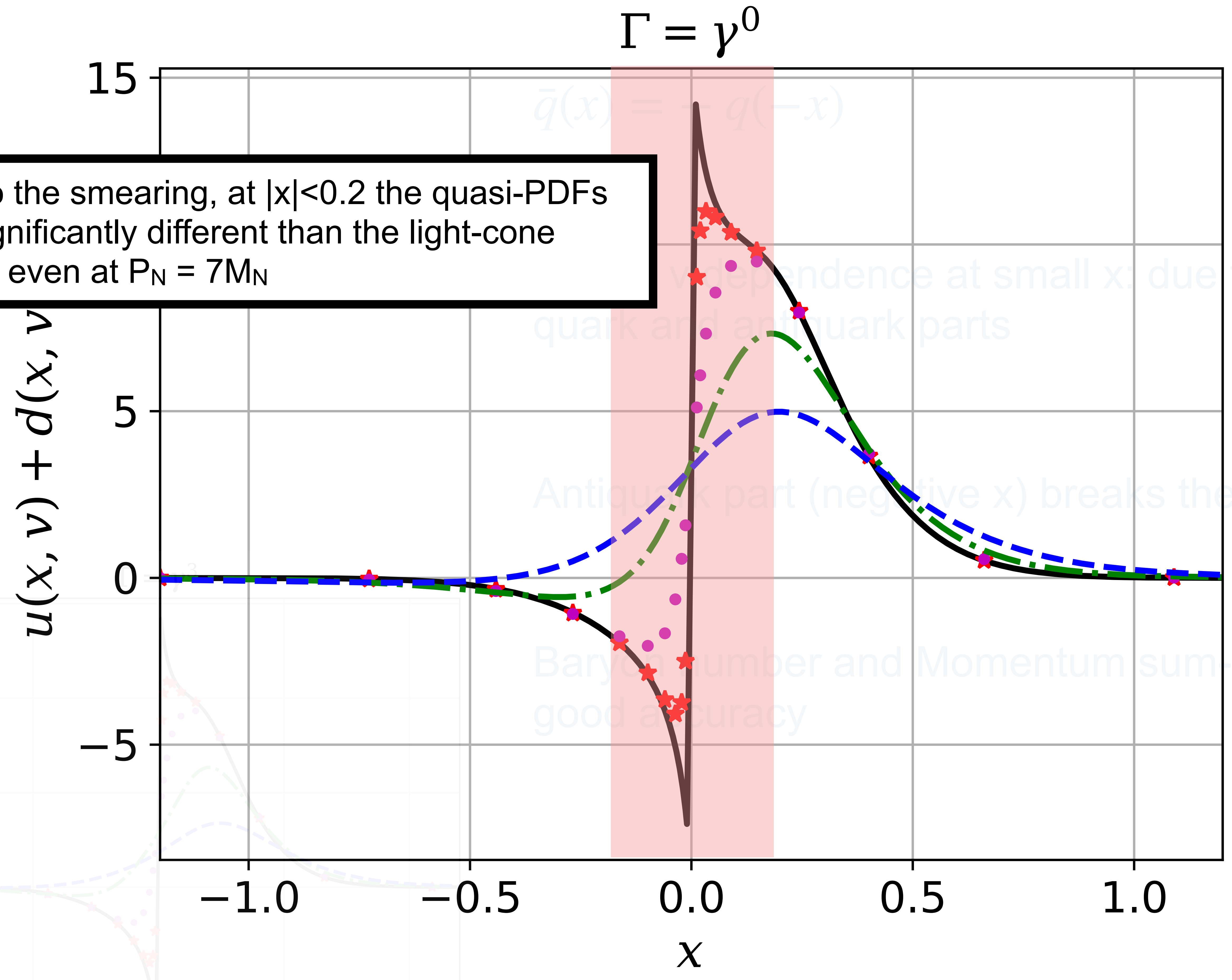
$u(x,v) + d(x,v)$

v=1	v=0.999	v=0.99	v=0.9	v=0.7
$P_N/M_N=\infty$	22.3	7.0	2.1	1.0



Due to the smearing, at |x|<0.2 the quasi-PDFs are significantly different than the light-cone PDFs, even at $P_N = 7M_N$

$\bar{q}(x) = -q(-x)$

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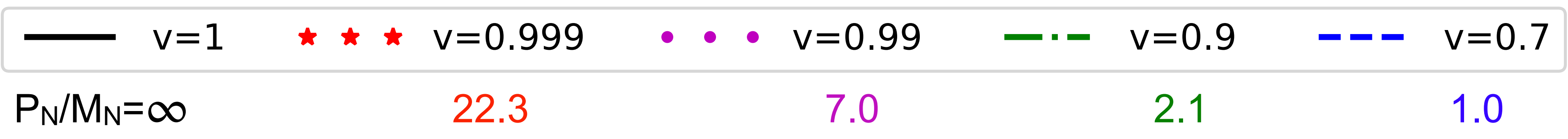
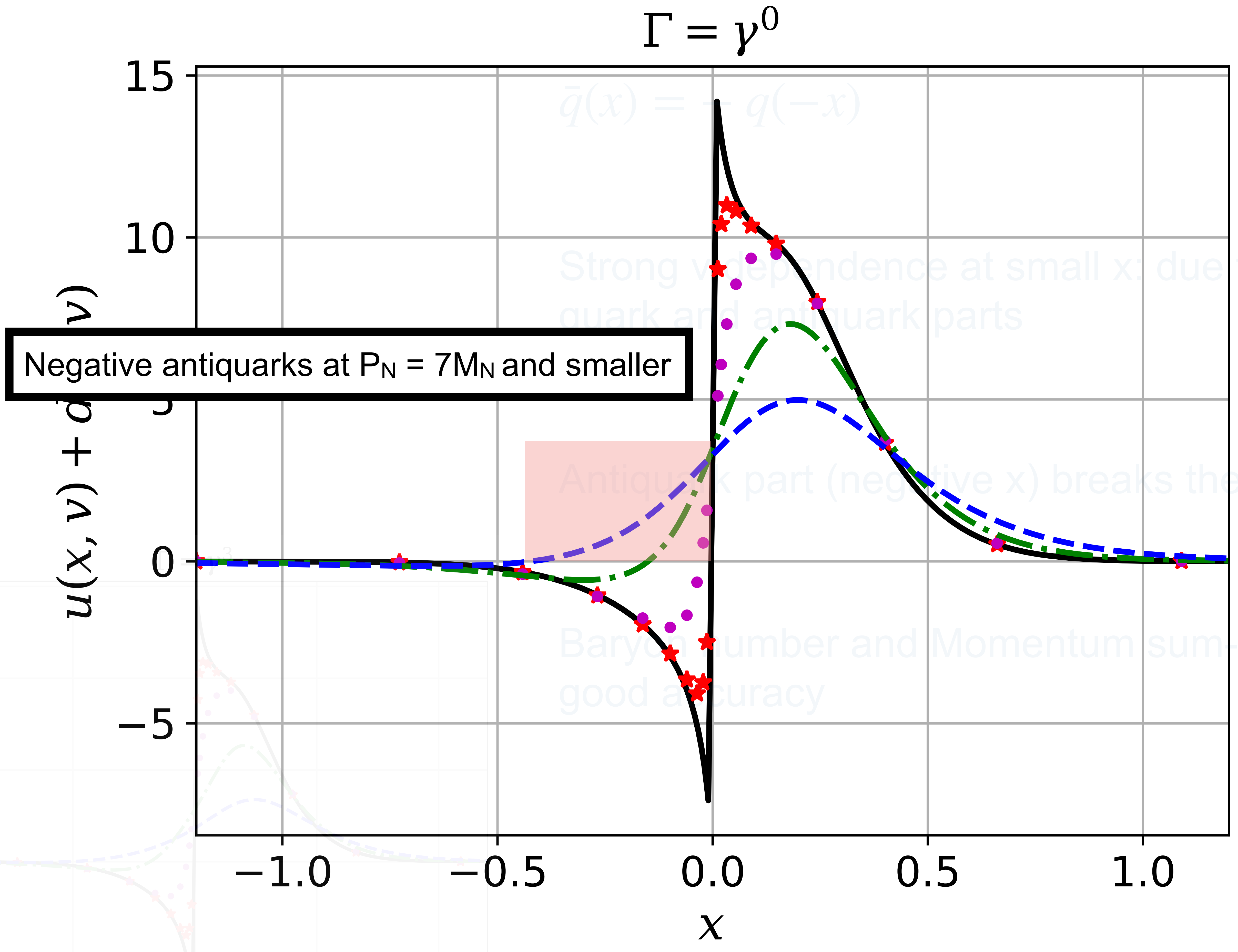
$u(x, v) + d(x, v)$





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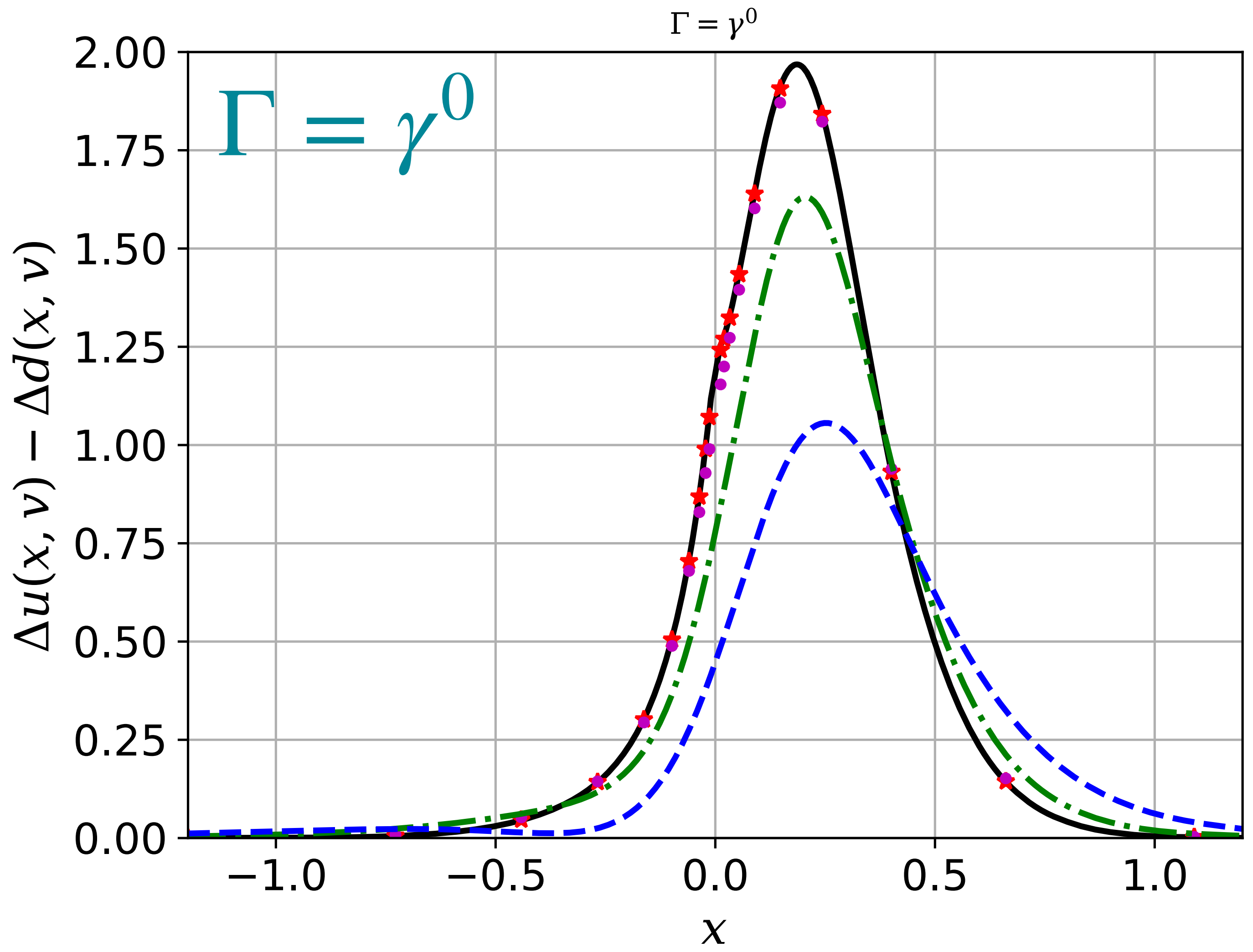
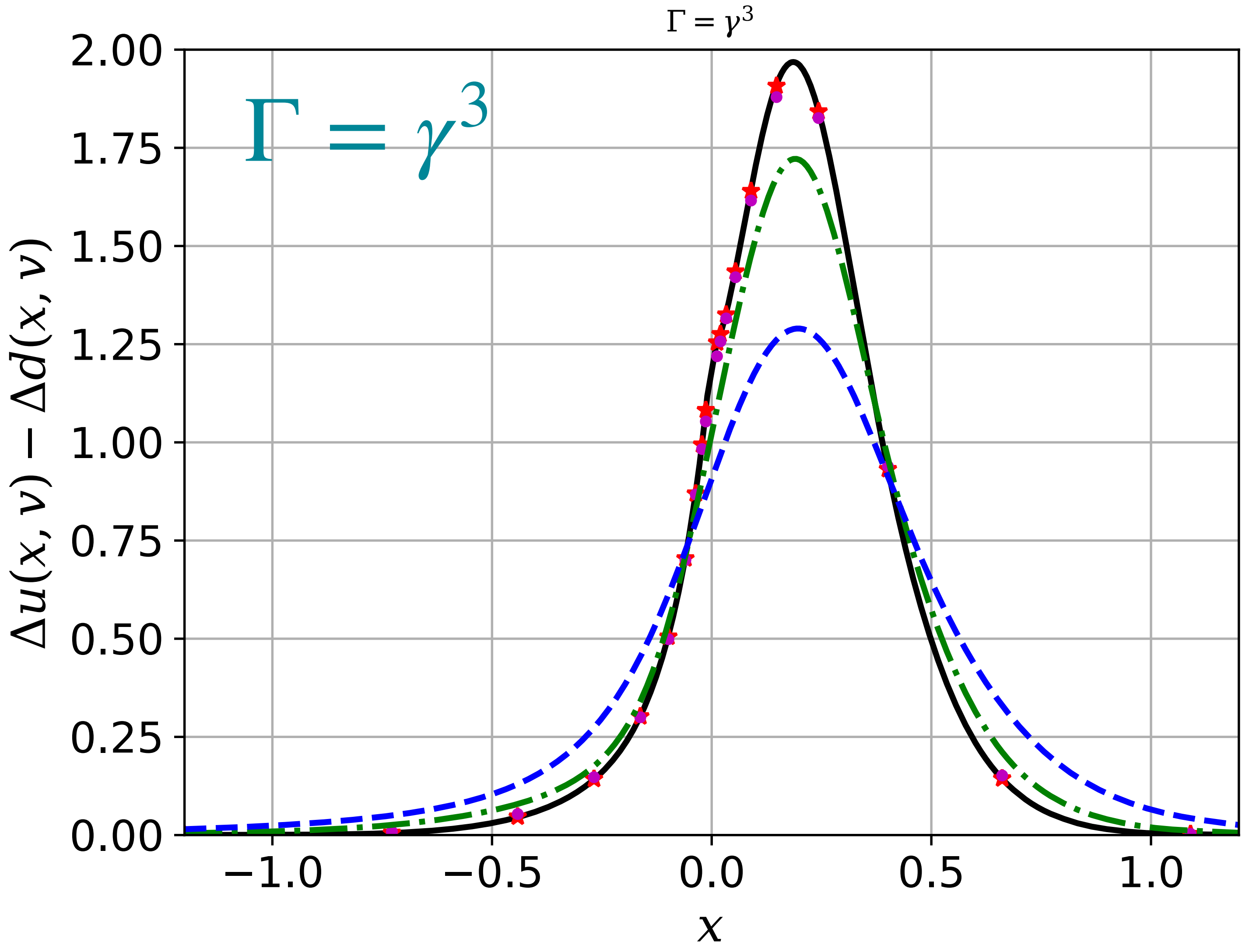




$\Delta\bar{q}(x) = \Delta q(-x)$

At v=0.9 (P ~ 2 GeV), qPDF ~ PDF

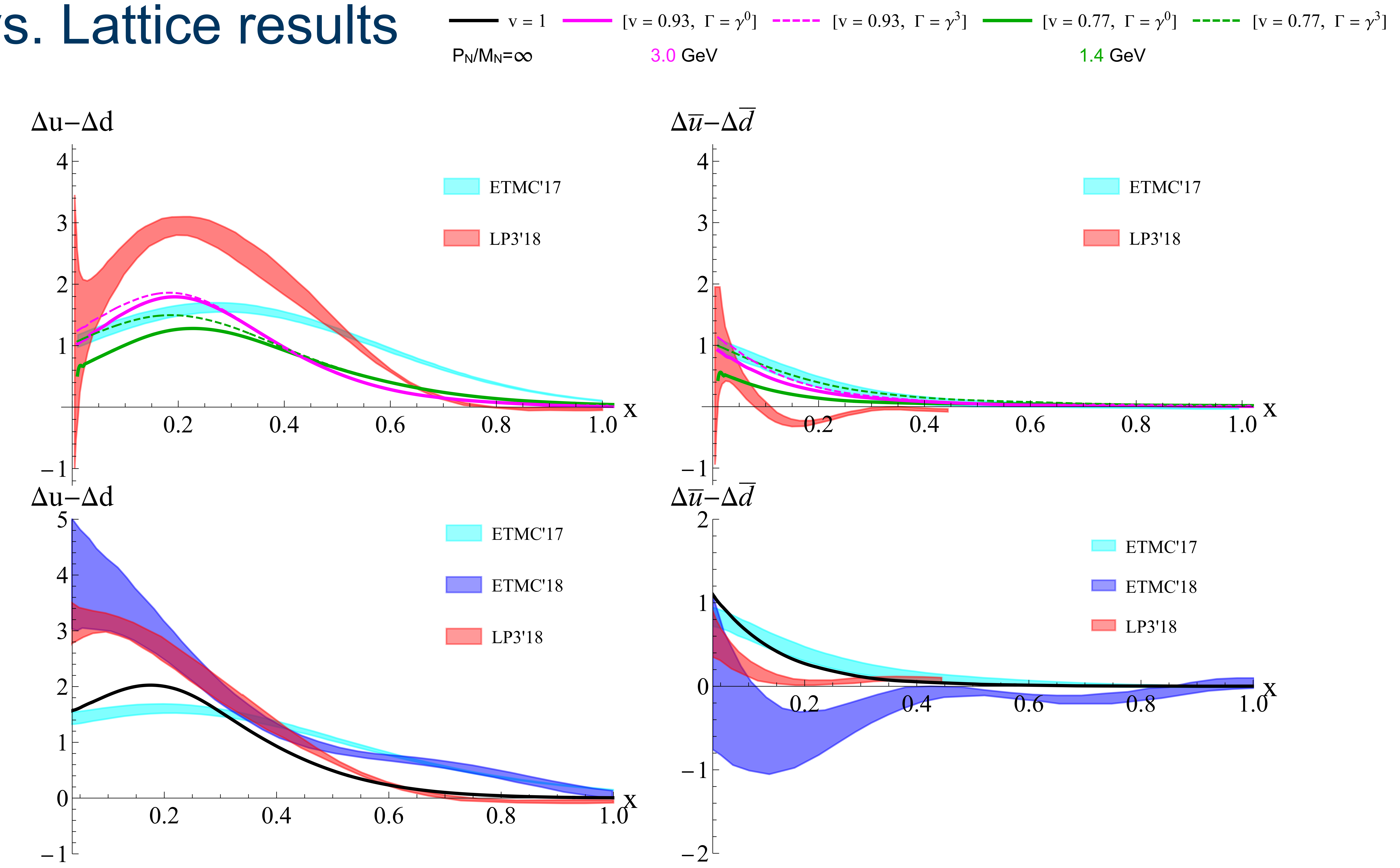
Sum-rules are satisfied in good accuracy

$\Gamma = \gamma^3$ qPDF converges faster to the lightcone PDF

$$\int_{-\infty}^{\infty} dx\, (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

[HDS, Phys.Lett.B 838 (2023) 137741]

vs. Lattice results



$(m_\pi, P_z, \mu) =$ (0.37, 1.4, 2.0) **[ETMC'17 Alexandrou et al. Phys. Rev. D, vol. 96, no. 1, p. 014513, 2017]**

(0.13,1.4,2.0) **[ETMC'18 Alexandrou et al. Phys.Rev.Lett. 121 (2018) 11, 112001, 2018]**

(0.135, 3.0, 3.0) **[LP3'18 Lin et al.Phys. Rev. Lett., vol. 121, no. 24, p. 242003, 2018]**

Antiquark flavor asymmetry

Antiquark asymmetries in the proton

Unpolarized antiquarks: $\bar{d} > \bar{u}$ [Glück, Reya, Vogt, ZPC (1995)]

PDFs from polarized DIS: assumed $\Delta\bar{u} - \Delta\bar{d} = 0$ [Glück, Reya, Volgesang, PLB 359 (1995)] [Glück et al., PRD 53 (1996)]

χQSM prediction: $\Delta\bar{u} - \Delta\bar{d}$ **is large and positive** [Diakonov et al., NPB (1996) / PRD (1997)]

DIS is insensitive to the antiquark flavor asymmetry, but Drell-Yan is! [Dressler et al, EPJC 14 (2000), EPJC 18 (2001)] [Kumano and Miyama, PLB 479 (2000)]

Analyses using DIS + SIDIS, Drell-Yan [Glück et al., PRD 63 (2001)] [De Florian et al, PRD 80 (2009)] [Nocera et al. (NNPDF), NPB 887 (2014)]

Single spin asymmetry (W-boson) in polarized PP collision is used to study the asymmetry

(STAR collaboration) [L. Adamczyk et al. PRL 113 (2014)] [A. Adare et al. PRD 98 (2018)] [J. Adam et al. PRD 99 (2019)]

Global analyses updates:

[De Florian et al. PRD 100 (2019)]
[Cocuzza et al. (JAM) arXiv:2202.03371 (2022)]

Antiquark asymmetries in the proton

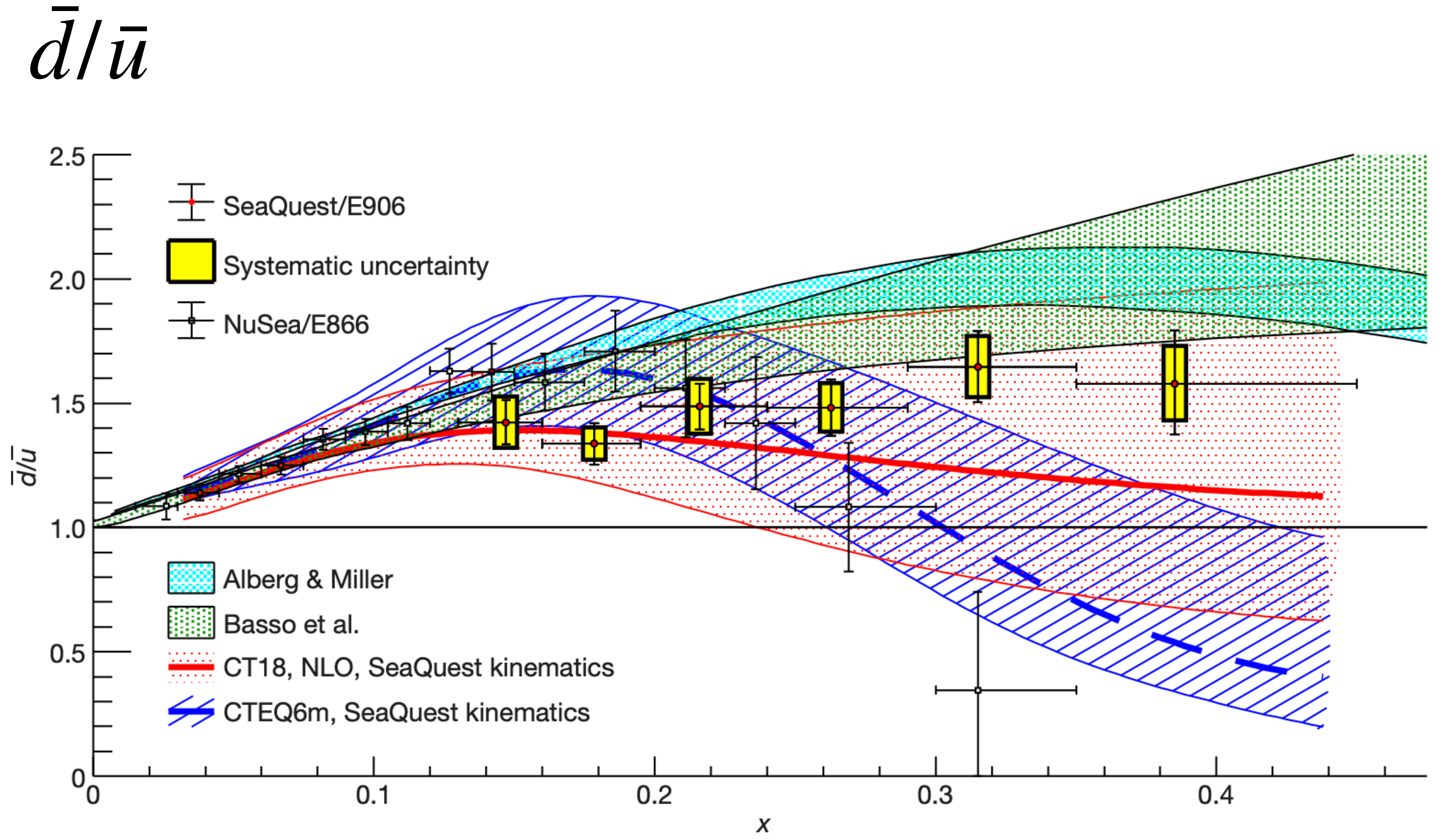


Fig. 2 | Ratios $\bar{d}(x)/\bar{u}(x)$. Ratios $\bar{d}(x)/\bar{u}(x)$ in the proton (red filled circles) with their statistical (vertical bars) and systematic (yellow boxes) uncertainties extracted from the present data based on NLO calculations of the Drell–Yan cross-sections. Also shown are the results obtained by the NuSea experiment (open black squares) with statistical and systematic uncertainties added in quadrature[4]. The cyan band shows the predictions of the meson–baryon model of Alberg & Miller[25] and the green band shows the predictions of the statistical parton distributions of Basso et al.[21]. The red solid (blue dashed) curves show the ratios $\bar{d}(x)/\bar{u}(x)$ calculated with CT18[29] (CTEQ6[35]) parton distributions at the scales of the SeaQuest results. The horizontal bars on the data points indicate the width of the bins.

[SeaQuest, Nature 590 (2021) 7847, 561-565]

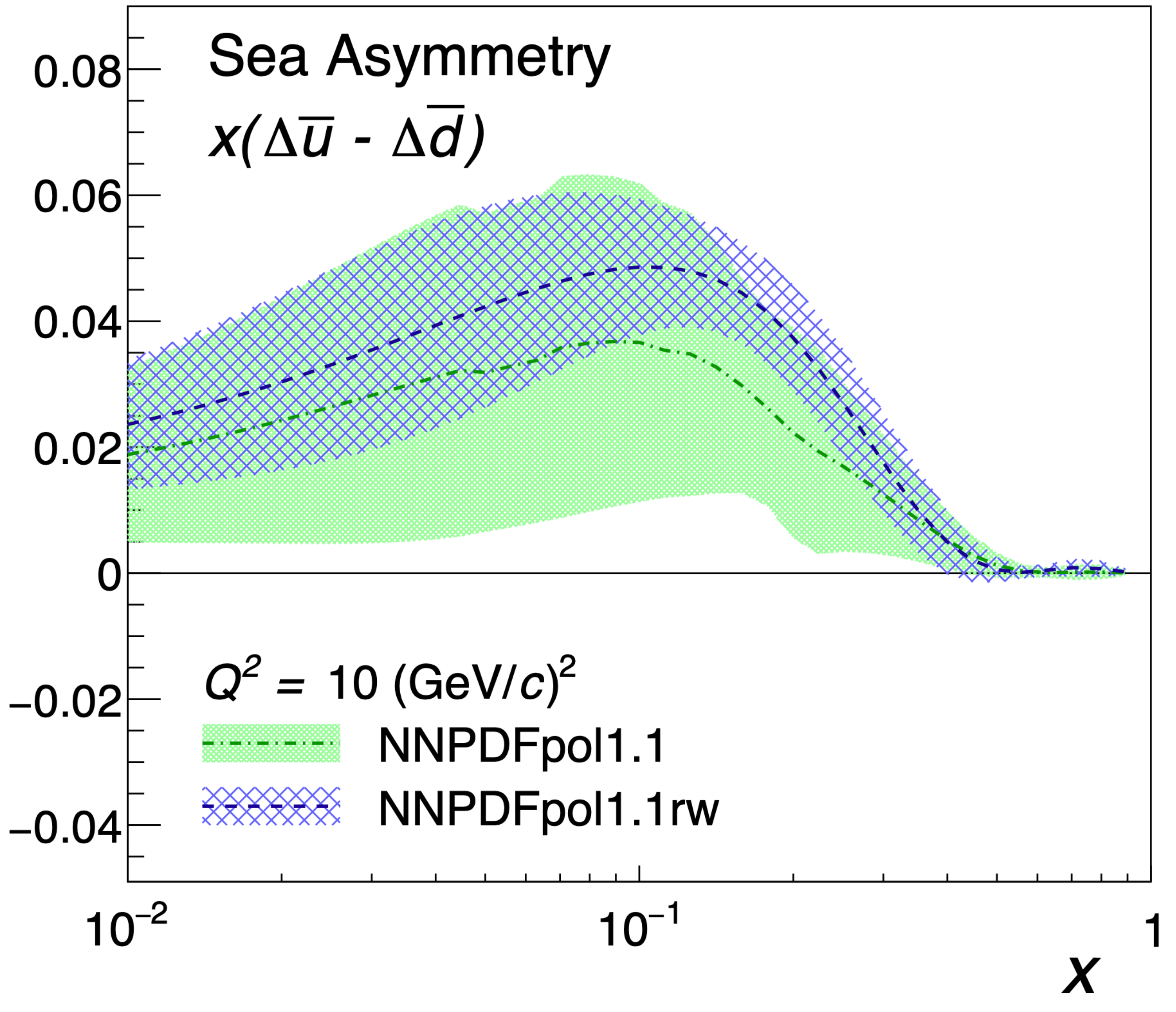
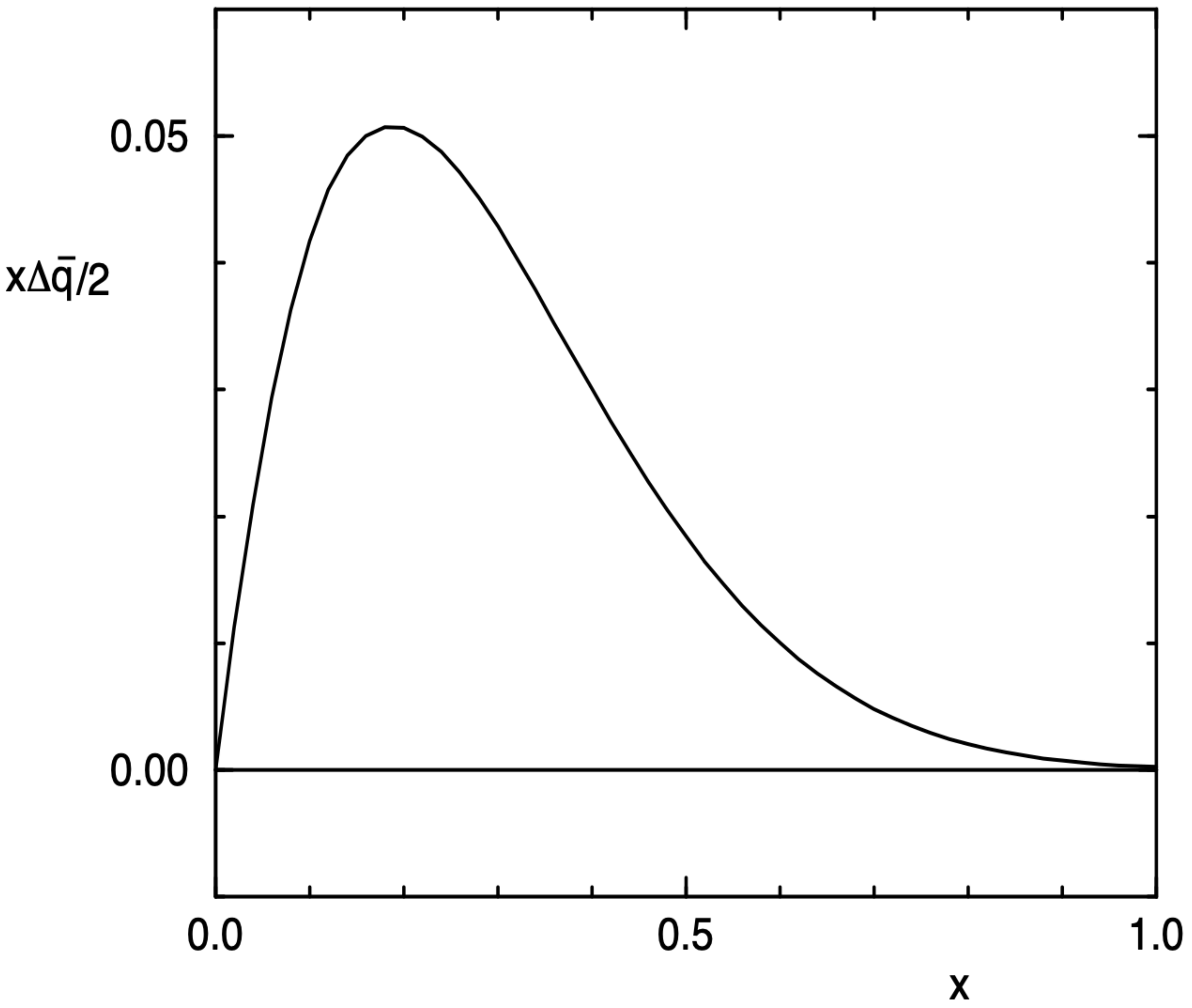


FIG. 6. The difference of the light sea-quark polarizations as a function of x at a scale of $Q^2 = 10\,(\mathrm{GeV}/c)^2$. The green band shows the NNPDFpol1.1 results [1] and the blue hatched band shows the corresponding distribution after the STAR 2013 $W^{\pm}$ data are included by reweighting.

[STAR collaboration, Phys.Rev.D 99 (2019) 5, 051102]



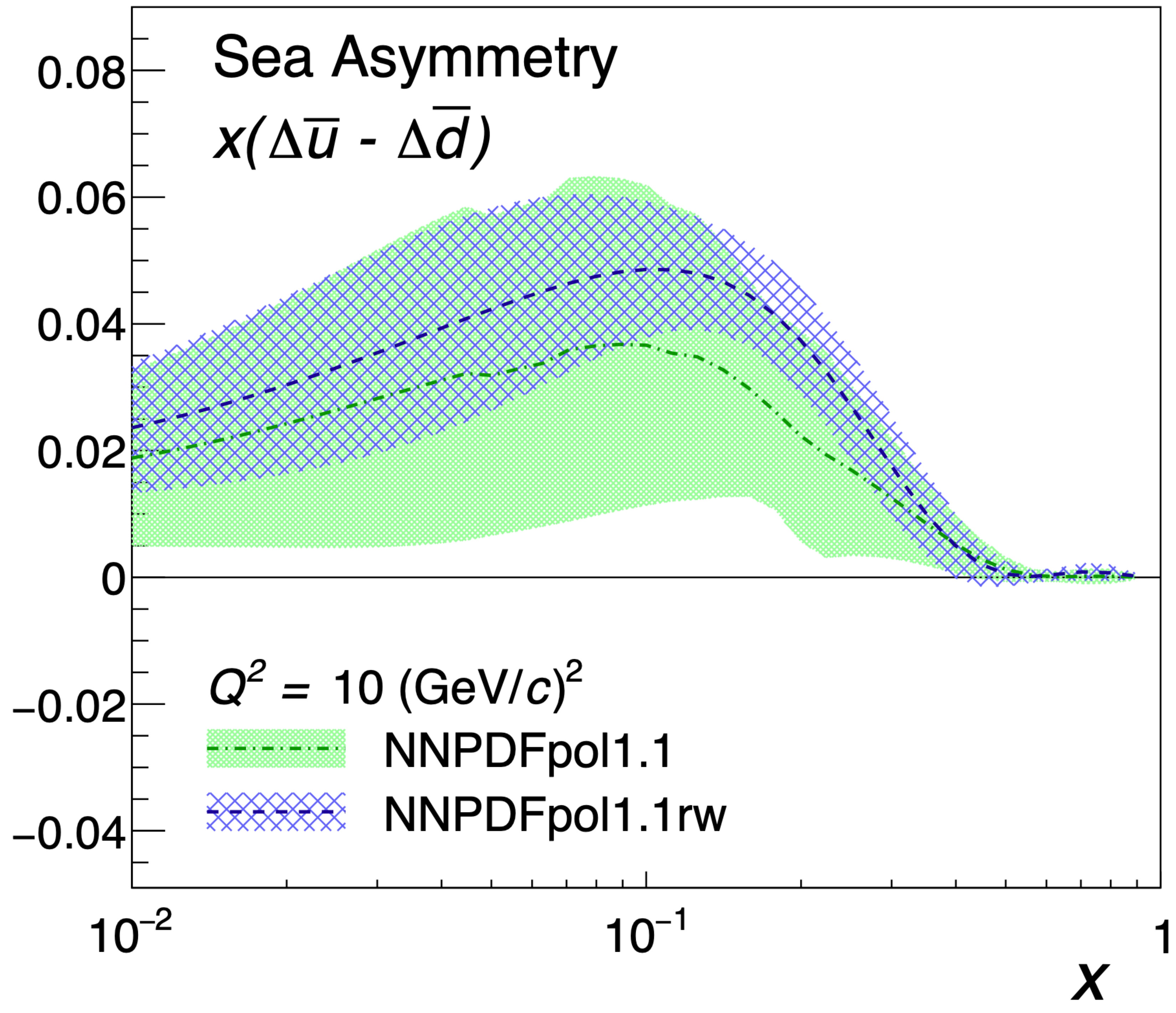
[Diakonov et al., NPB (1996) / PRD (1997)]

Figure 6: The isovector polarized antiquark distribution, $\frac{1}{2}x[\Delta\bar{u}(x) - \Delta\bar{d}(x)]$. *Solid line:* calculated distribution (total result, *cf.* Fig.2). In the fit of ref.[4] this distribution is assumed to be zero.

[ref.[4] Glück et al., PRD 53 (1996)]

Polarized antiquark flavor asymmetry: model case

[HDS, A. Tandogan, in preparation]



[STAR collaboration, Phys.Rev.D 99 (2019) 5, 051102]

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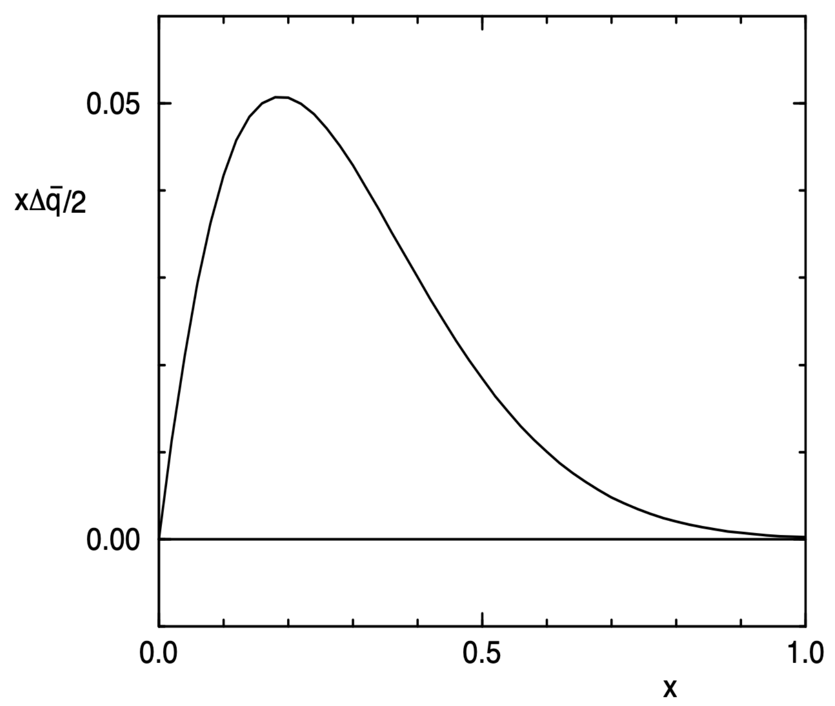

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[Diakonov et al., NPB (1996) / PRD (1997)]

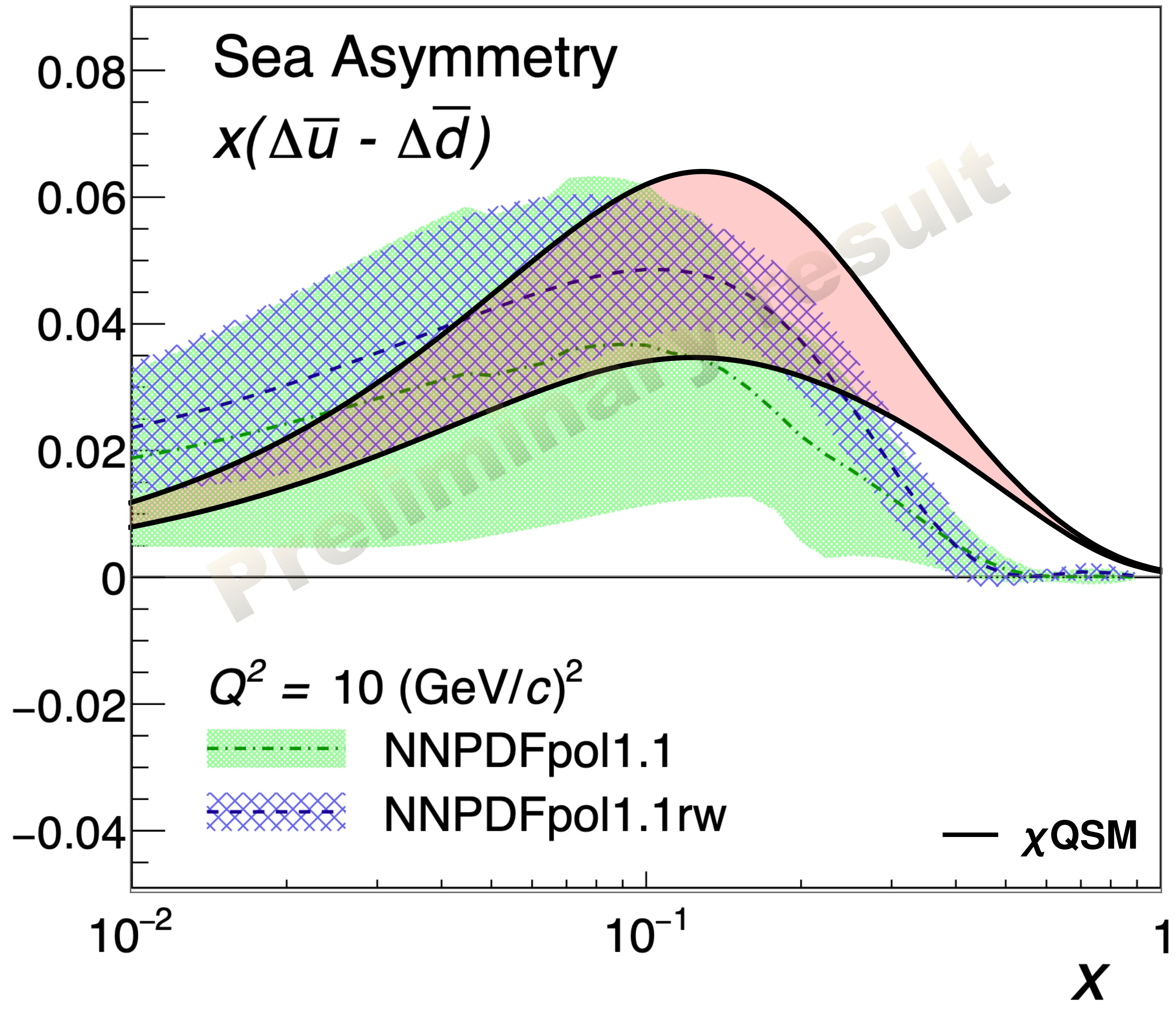
M = 350 MeV

Model allows the following parameter window, depending on ρ/R with fixed ρ

M [MeV]	330	420
M_N [MeV]	1161	1077
ρ/R	0.32	0.37
F_π [MeV]	77	90

Polarized antiquark flavor asymmetry: model case

[HDS, A. Tandogan, in preparation]



[STAR collaboration, Phys.Rev.D 99 (2019) 5, 051102]

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Band: Model systematic uncertainty

Scale: ρ~1/(600MeV), in the chiral limit

M [MeV]	330	420
M_N [MeV]	1161	1077
ρ/R	0.32	0.37
F_π [MeV]	77	90

Continuum contribution (Polarized vacuum) is crucial

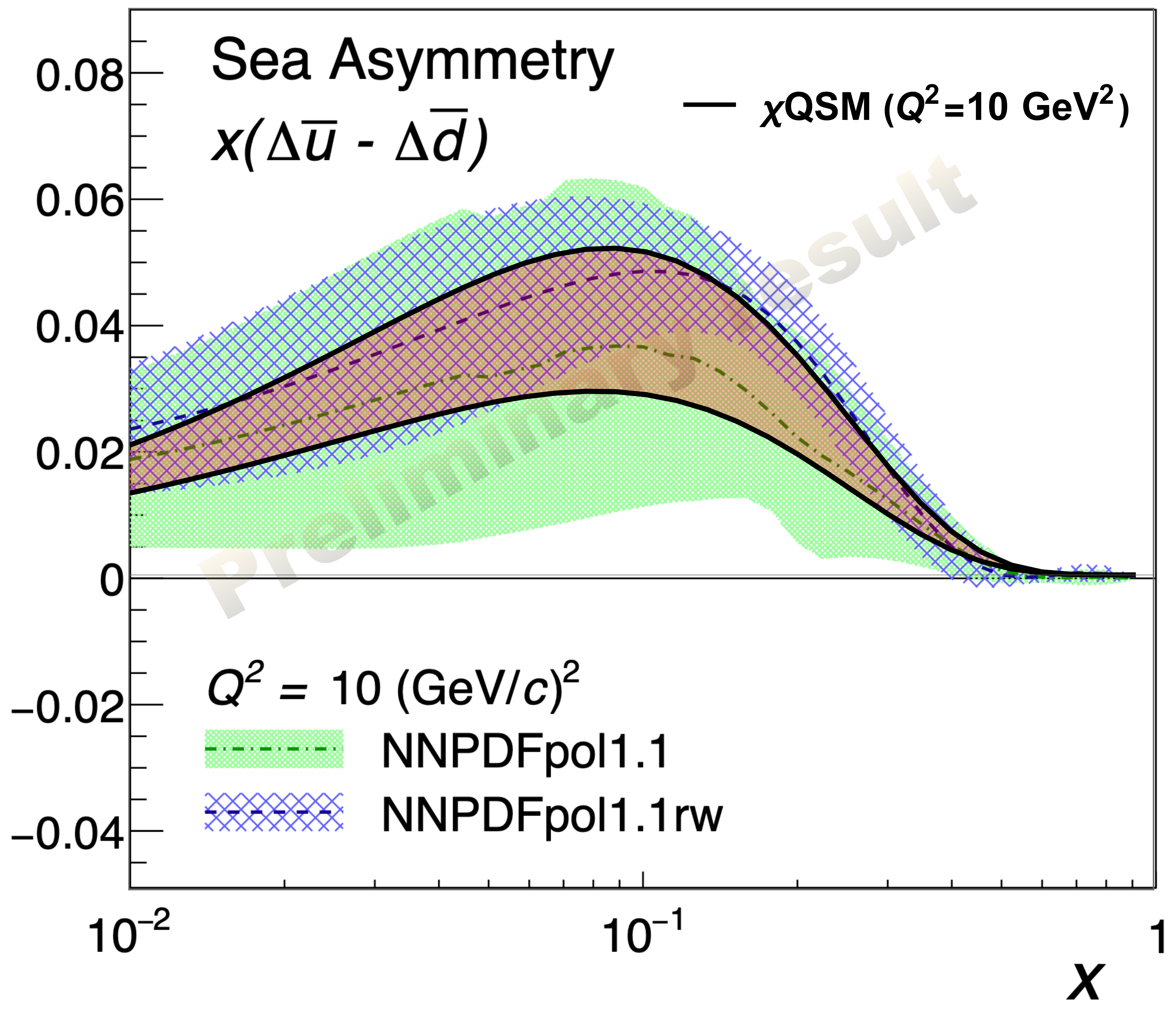
? 1/Nc correction can enhance the PDF ~30%

? Hardness: quark virtuality (momentum dep. quark mass)

? Scale evolution

Polarized antiquark flavor asymmetry: model case

[HDS, A. Tandogan, in preparation]



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Scale evolution

Fitted with $x\Delta\bar{u} = Nx^a(x-1)^b$

Large Nc initial condition: $\Delta\bar{u} + \Delta\bar{d} = 0$

NLO DGLAP evolution (HOPPET package)

Fixed initial scale ρ = 1/(600MeV)

Upper band M=330MeV

Closing remarks

Summary

- χQSM provides a reasonable description on the (quasi-)PDFs at low scale
- Sum-rules for quasi-PDFs depend on their definitions
  $\bar{c}^q$, 'better' Γ for the convergence to the PDFs
- Good convergence of the $\Delta u - \Delta d$ to the light-cone PDF
  vs. $u + d$, at small x, obviously Pz=3GeV is not enough!

Future tasks

Small pdfs in the large Nc (Singlet distributions & Flavour asymmetries)

More realistic model: quark virtuality from the instantons

$\rightarrow$ momentum dependent quark mass

$\rightarrow$ necessary to describe the GPDs

$\rightarrow$ under development by Yongwoo Choi (Inha)

Including the gluon explicitly

$\rightarrow$ Gluon structure functions, EMT form factors, higher twist, …

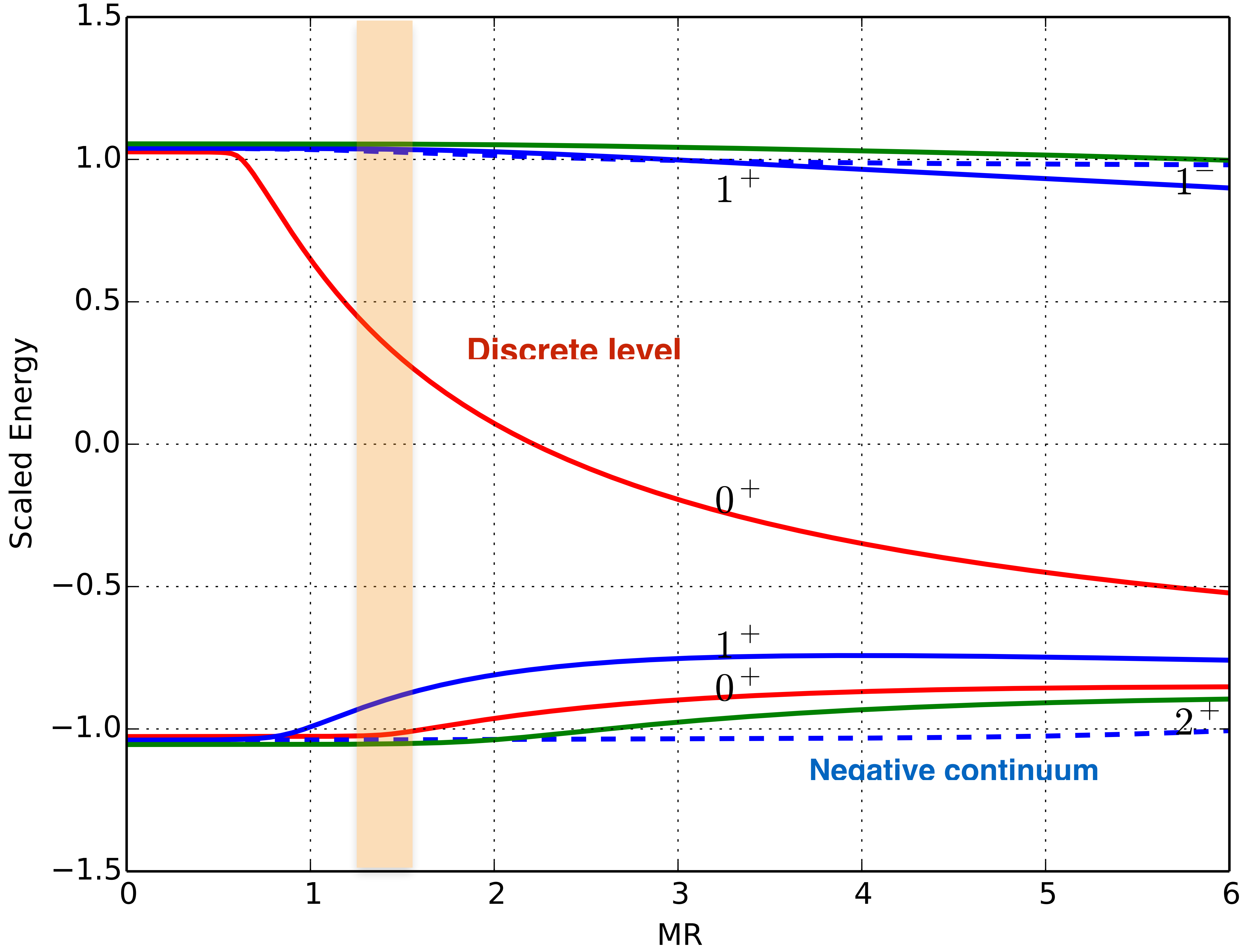
$\rightarrow$ The mass & spin decomposition of the proton

Flavour SU(3)

Thank you very much!

Backup slides

Hedgehog Ansatz: $U_{\mathrm{SU}(2)} = \exp\left[i\gamma_5 \mathbf{n}\cdot\boldsymbol{\tau} P(r)\right]$



Quantum Numbers:

G = J + τ

P = $(-1)^{G,G+1}$

Quarks are bound by the pion mean-field

Numerical calculation

Ansatz for the pion meanfield

[D. Diakonov, V. Petrov, and P. Pobylitsa, Nucl. Phys. B 306, 809 (1988)]

$$P(r) = 2\,\mathrm{Arctan}\left(\frac{r_0^2}{r^2}\right) \qquad r_0 \approx 1/M$$

within ~10% from the self-consistent solution

Interpolation formula

$$\frac{pM}{p^2+M^2}(U-1) \ll 1$$

Quasi-PDFs have the same order of divergence as the PDFs (v=1)

with smooth convergence in v →1

Logarithmic divergence: Pauli-Villars regularization

$$q(x,v)^{PV} = q(x,v)^{\text{level}} + q(x,v)_{occ} - \frac{M^2}{M_{PV}^2} q(x,v)_{occ}(M \to M_{PV})$$

$$F_\pi^2 = \frac{N_c M^2}{4\pi^2}\log(M_{PV}^2/M^2)$$

$$M = 350 \text{ MeV}$$
$$M_{PV} = 557 \text{ MeV}$$

Quasi-PDFs in the χQSM

Nucleon at rest → Lorentz boost to a inertial frame with velocity v in the z direction

Quark and antiquark quasi number densities $x \in (-\infty, \infty)$

$$D_f(x,v) = \frac{1}{2E_N}\int \frac{d^3k}{(2\pi)^3}\delta\left(x - \frac{k^3}{P_N}\right)\int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}}\langle N_v|\bar{\psi}_f\left(-\frac{\mathbf{x}}{2},t\right)\Gamma\psi_f\left(\frac{\mathbf{x}}{2},t\right)|N_v\rangle$$

$$\bar{D}_f(x,v) = \frac{1}{2E_N}\int \frac{d^3k}{(2\pi)^3}\delta\left(x - \frac{k^3}{P_N}\right)\int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}}\langle N_v|\mathrm{Tr}\left[\Gamma\psi_f\left(-\frac{\mathbf{x}}{2},t\right)\bar{\psi}_f\left(\frac{\mathbf{x}}{2},t\right)\right]|N_v\rangle$$

become exact number density in the limit $v \to \infty$

Both the $\Gamma = \gamma^0$ and $\Gamma = \gamma^3$ define quasi-PDFs

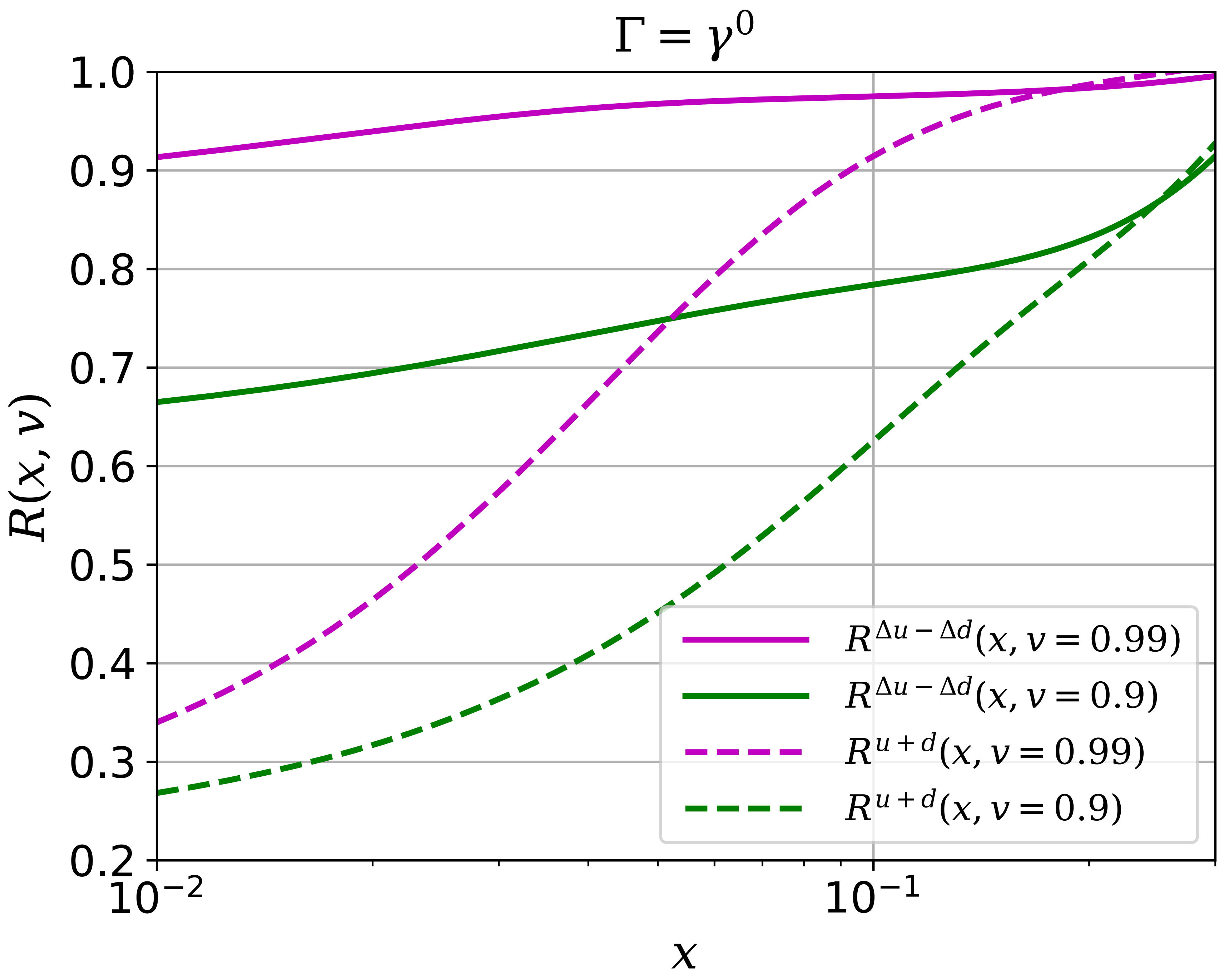
A representation for the Green's function in the χQSM

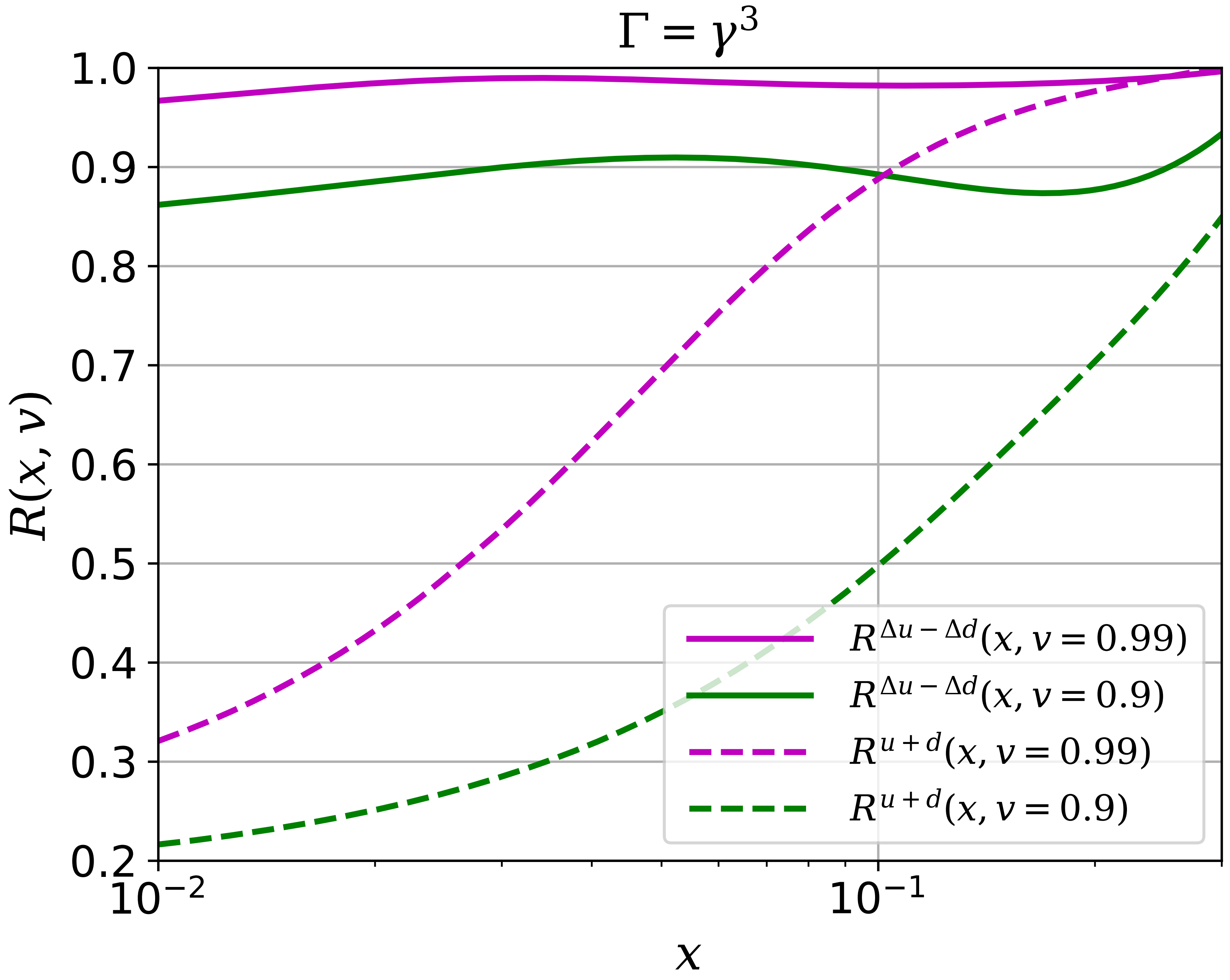
$$\langle N_v|\mathrm{T}\left\{\psi(\vec{x}_1,t_1)\bar{\psi}(\vec{x}_2,t_2)\right\}|N_v\rangle = -\,S[\vec{v}]\left[\Theta(t_2-t_1)\sum_{occ}\Phi_n(\vec{x}_1)\Phi_n^\dagger(\vec{x}_2)\gamma_0\exp(-iE_n(t_1-t_2))\right.$$
$$\left.-\Theta(t_1-t_2)\sum_{nocc}\Phi_n(\vec{x}_1)\Phi_n^\dagger(\vec{x}_2)\gamma_0\exp(-iE_n(t_1-t_2))\right]S^{-1}[\vec{v}]$$

IVP vs ISU

v=1 | v=0.999 | v=0.99 | v=0.9 | v=0.7

$P_N/M_N=\infty$ | 22.3 | 7.0 | 2.1 | 1.0





$$R^q(x, v) \equiv q(x, v)/q(x, v = 1)$$

IVP vs ISU

v=1 | v=0.999 | v=0.99 | v=0.9 | v=0.7

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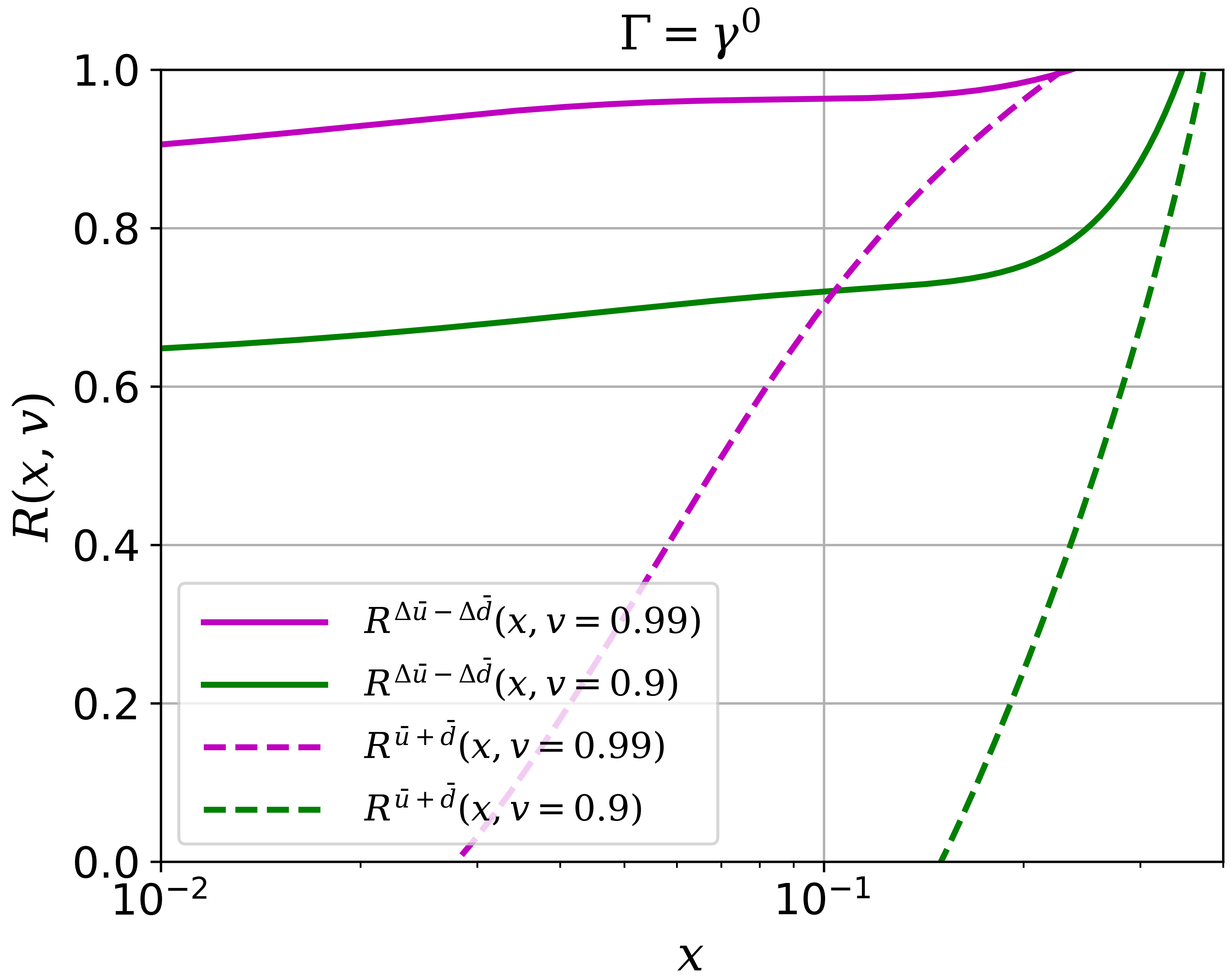
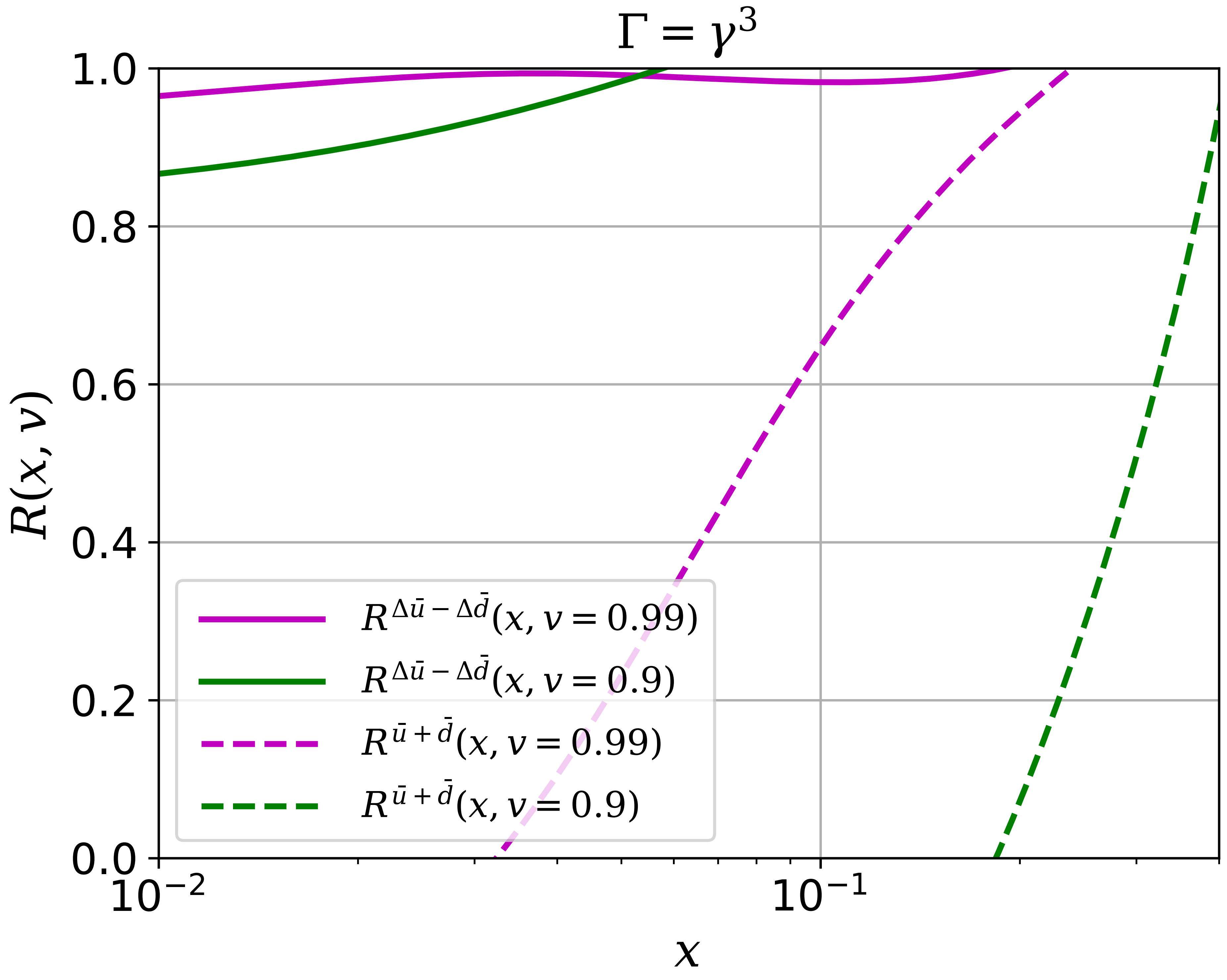


$$R^q(x, v) \equiv q(x, v)/q(x, v = 1)$$



Copied from [STAR collaboration, Phys.Rev.D 99 (2019) 5, 051102]

The leptonic $W^+ \to e^+\nu$ and $W^- \to e^-\bar{\nu}$ decay channels provide sensitivity to the helicity distributions of the quarks, Δu and Δd, and antiquarks, $\Delta\bar{u}$ and $\Delta\bar{d}$, that is free of uncertainties associated with non-perturbative fragmentation. The cross-sections are well described [18]. The primary observable is the longitudinal single-spin asymmetry $A_L \equiv (\sigma_+ - \sigma_-)/(\sigma_+ + \sigma_-)$ where $\sigma_{+(-)}$ is the cross-section when the helicity of the polarized proton beam is positive (negative). At leading order,

$$A_L^{W^+}(y_W) \propto \frac{\Delta\bar{d}(x_1)u(x_2) - \Delta u(x_1)\bar{d}(x_2)}{\bar{d}(x_1)u(x_2) + u(x_1)\bar{d}(x_2)}, \quad (1)$$

$$A_L^{W^-}(y_W) \propto \frac{\Delta\bar{u}(x_1)d(x_2) - \Delta d(x_1)\bar{u}(x_2)}{\bar{u}(x_1)d(x_2) + d(x_1)\bar{u}(x_2)}, \quad (2)$$

where x_1 (x_2) is the momentum fraction carried by the colliding quark or antiquark in the polarized (unpolarized) beam. $A_L^{W^+}$ ($A_L^{W^-}$) approaches $-\Delta u/u$ ($-\Delta d/d$) in the very forward region of W rapidity, $y_W \gg 0$, and $\Delta\bar{d}/\bar{d}$ ($\Delta\bar{u}/\bar{u}$) in the very backward region of W rapidity, $y_W \ll 0$. The observed positron and electron pseudorapidities, η_e, are related to y_W and to the decay angle of the positron and electron in the W rest frame [19]. Higher-order corrections to $A_L(\eta_e)$ are known [20–22] and have been incorporated into the aforementioned global analyses.

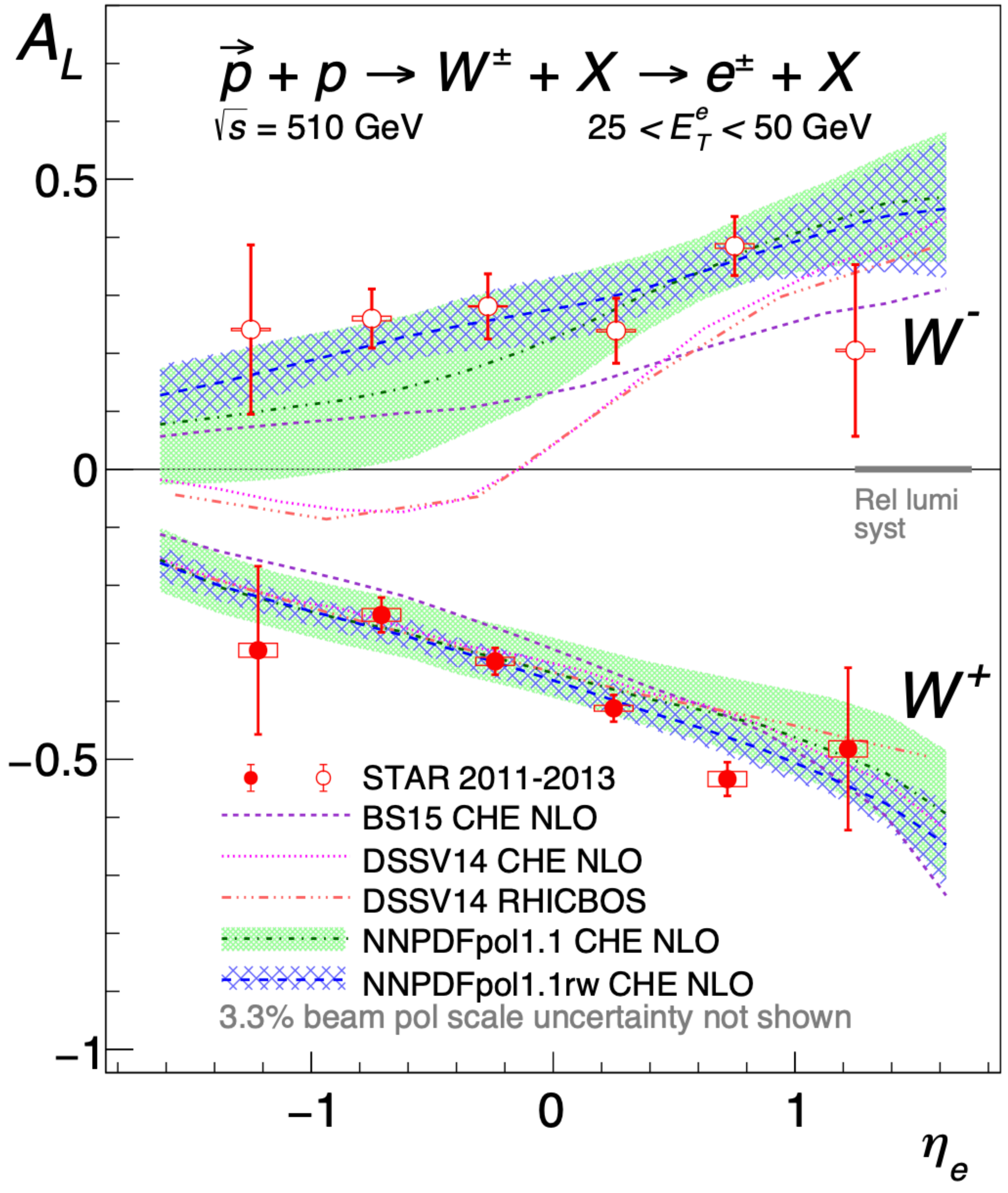
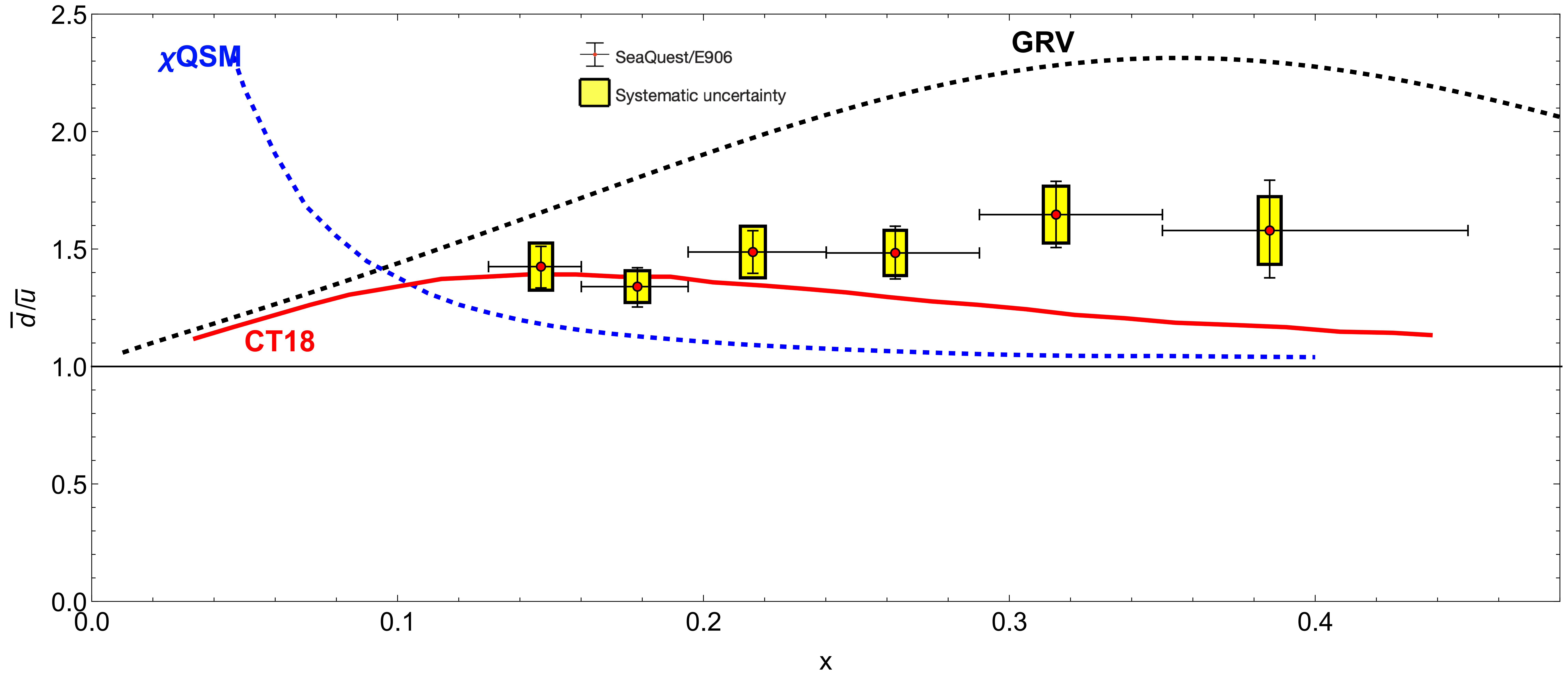


FIG. 5. Longitudinal single-spin asymmetries, A_L, for $W^\pm$ production as a function of the positron or electron pseudorapidity, η_e, for the combined STAR 2011+2012 and 2013 data samples for $25 < E_T^e < 50$ GeV (points) in comparison to theory expectations (curves and bands) described in the text.

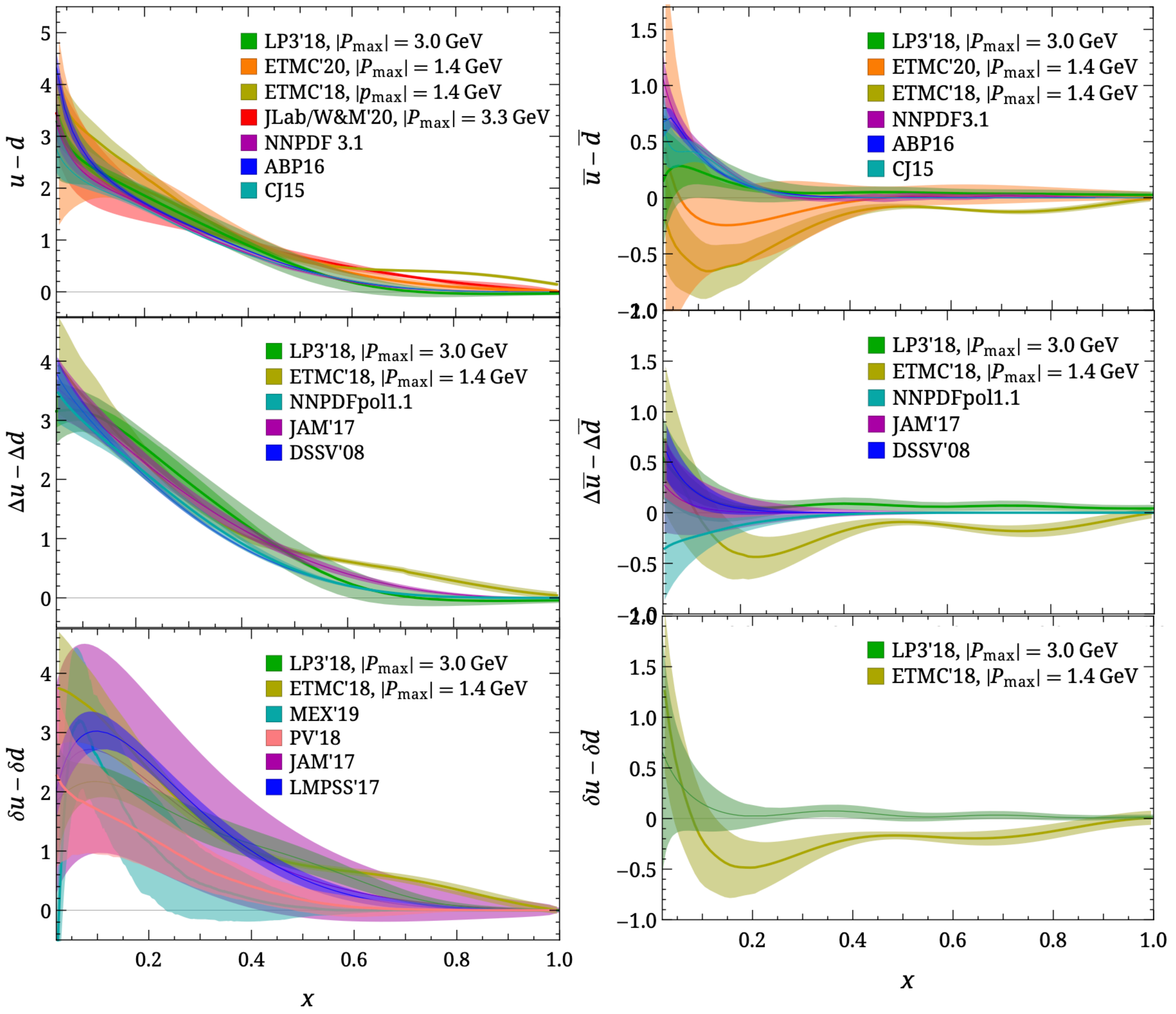


χQSM
SeaQuest/E906
Systematic uncertainty
GRV
CT18
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.1
0.2
0.3
0.4
$\bar{d}/\bar{u}$
x

Isovector PDFs

M. Constantinou's slide @ Spin 2021, Japan

State-of-the-art results



No continuum extrapolation yet

M. Constantinou *et al.* (2020) 2007.08636