

Lecture II

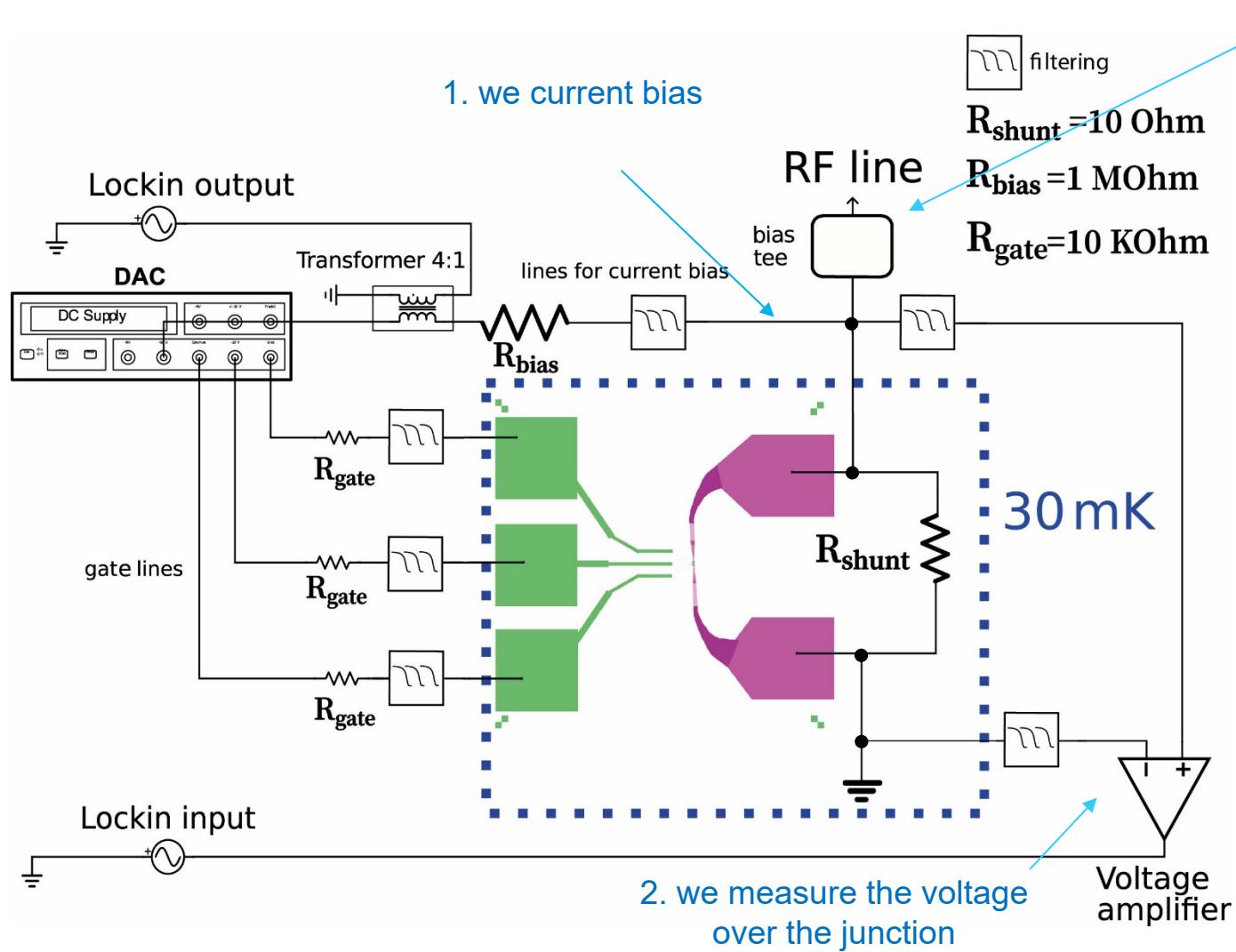
1st reminder from last lecture

Bias circuitry



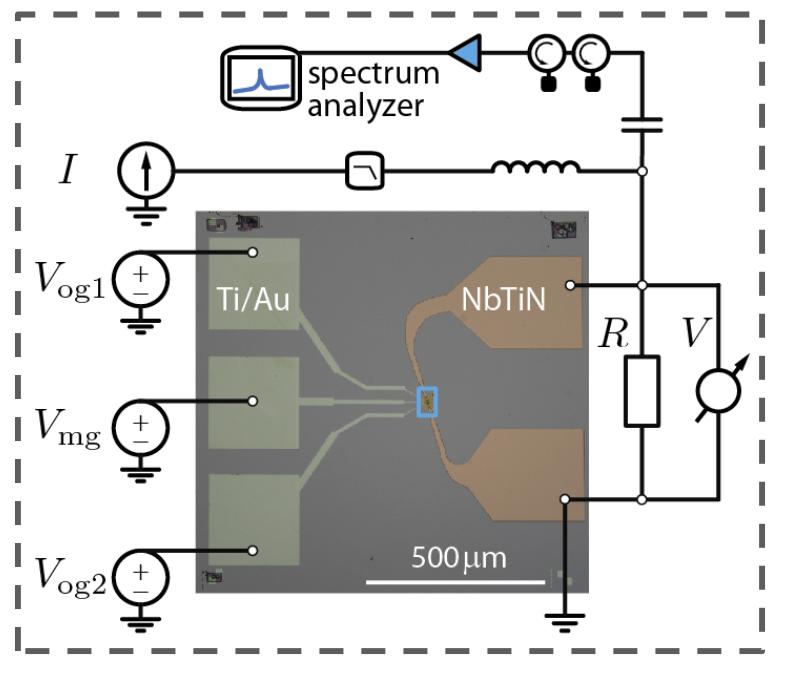
To wrap-up

For Josephson radiation we use a circuit similar to a Shapiro measurement. It consists of a JJ with a **proper shunt resistor** allowing to apply a DC bias over the junction:

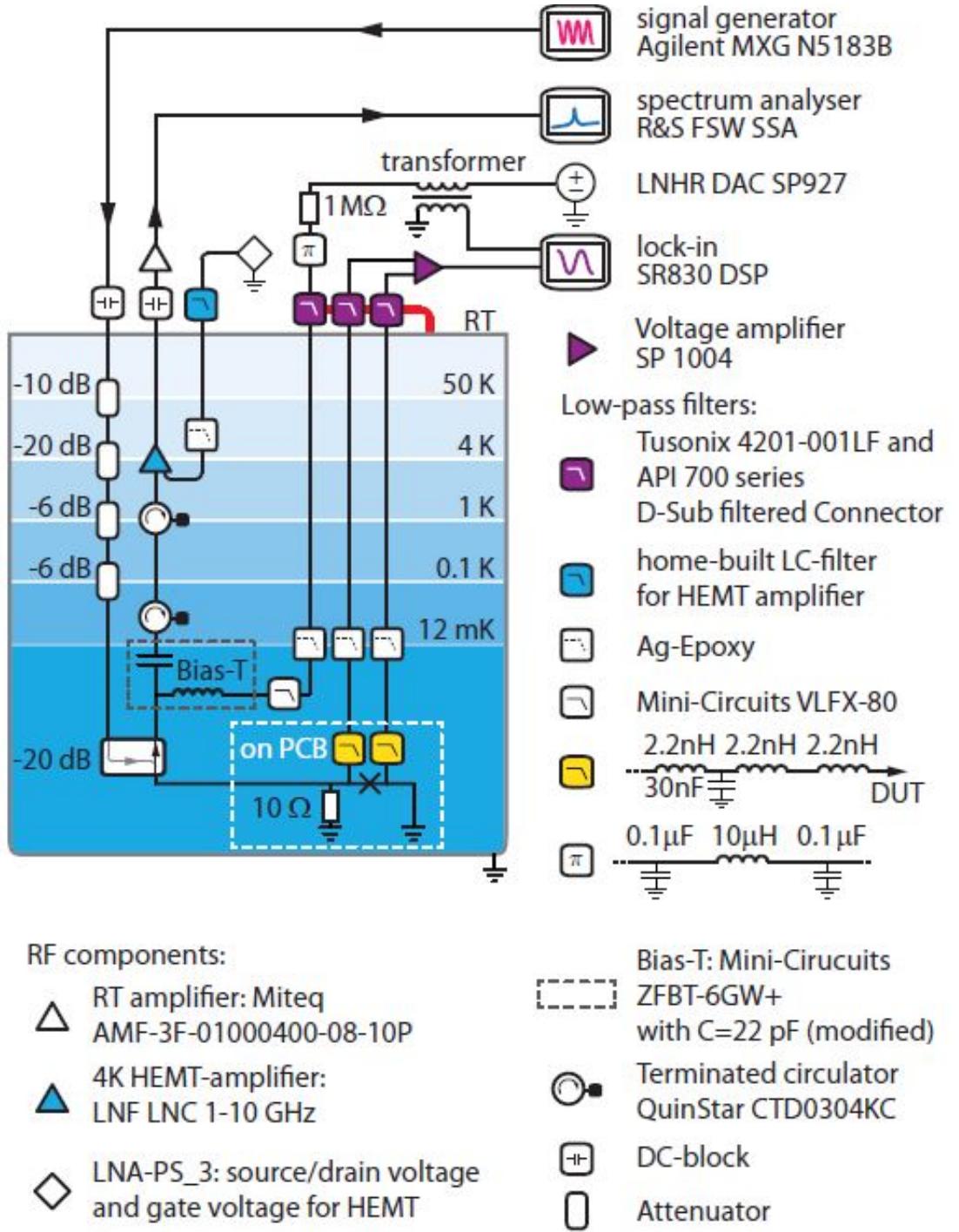


and 3. we also measure the RF radiation using cryogenic low noise wide bandwidth amplifiers → Josephson rad.

Cryogenic setup



The fractional Josephson effect: Current-Phase Relation



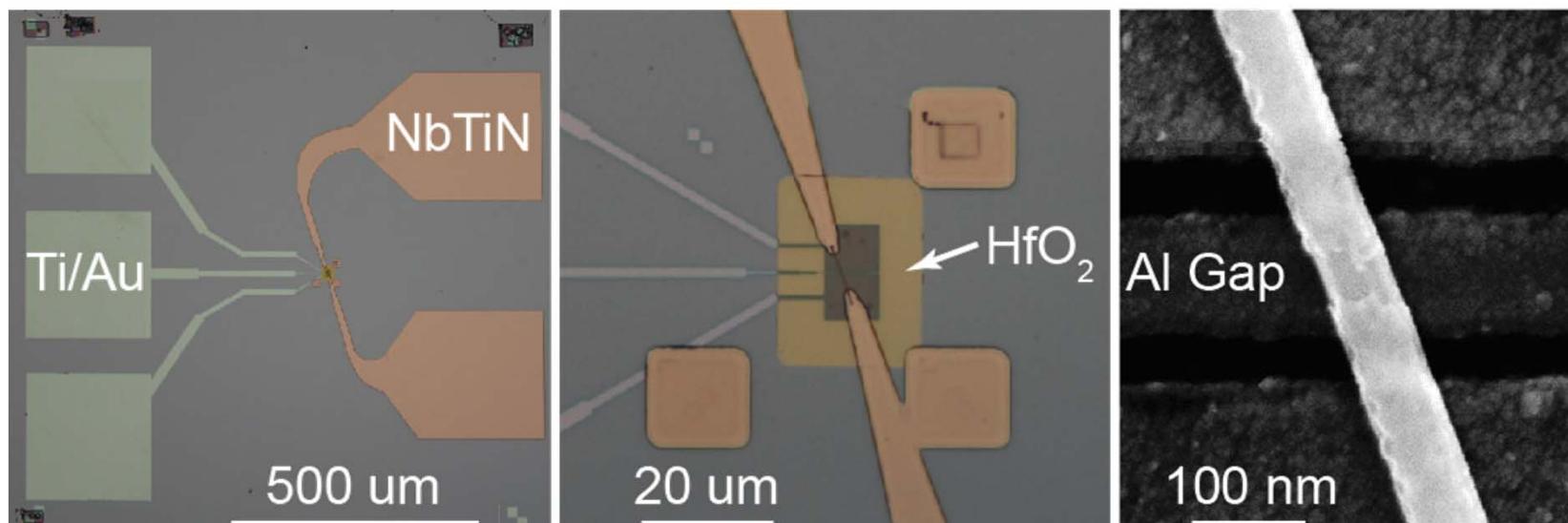
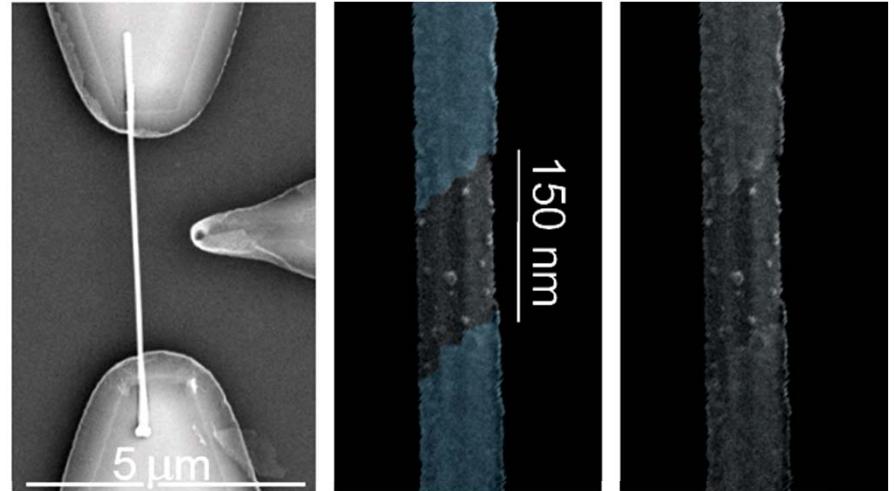
Al-InAs-NW-Al as reference



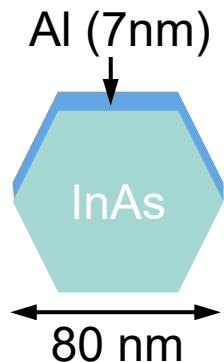
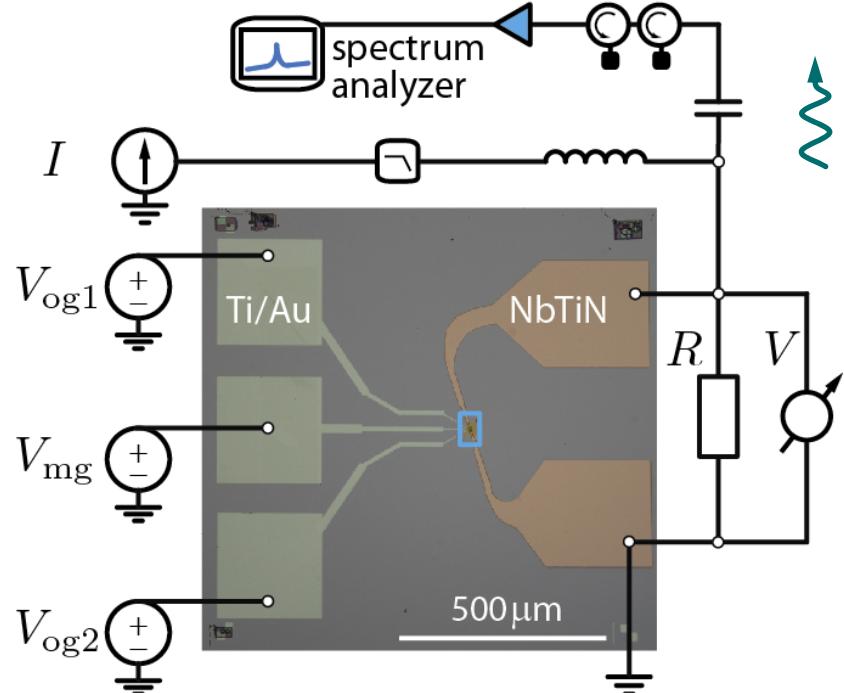
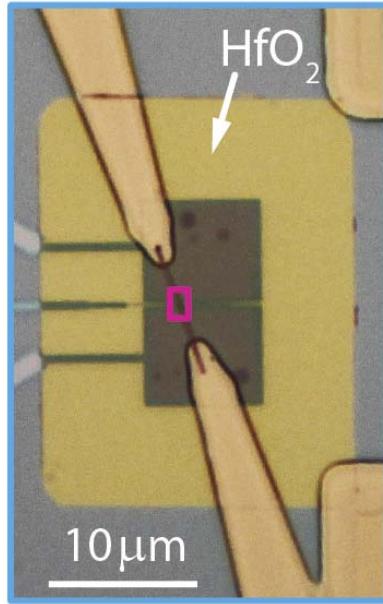
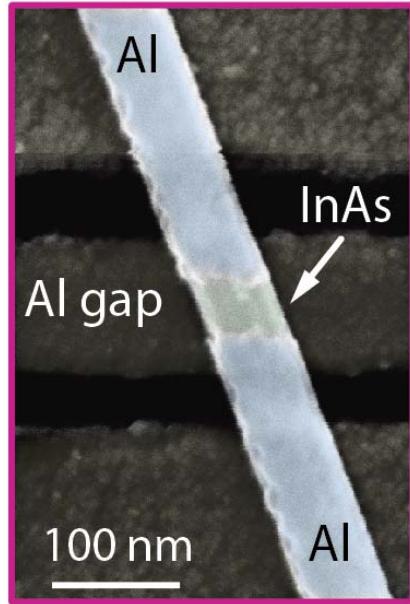
We have tested our setup sensitivity also for **InAs nanowires** with few modes and show that we can still detect the **Josephson radiation for a supercurrent that is due to a single conducting channel**.

This slide first shows the nanowires we use. They are from Copenhagen, so called “half-shell-coated” InAs NWs with Al as shell.

The figures on the right show a typical device where the Al shell is indicated with a weak blue touch. Here there is one gate from the right. In the device in which we have measured Josephson radiation, there are three gates



Al-InAs-NW-Al as reference



InAs nanowire with epitaxial Al half-shell

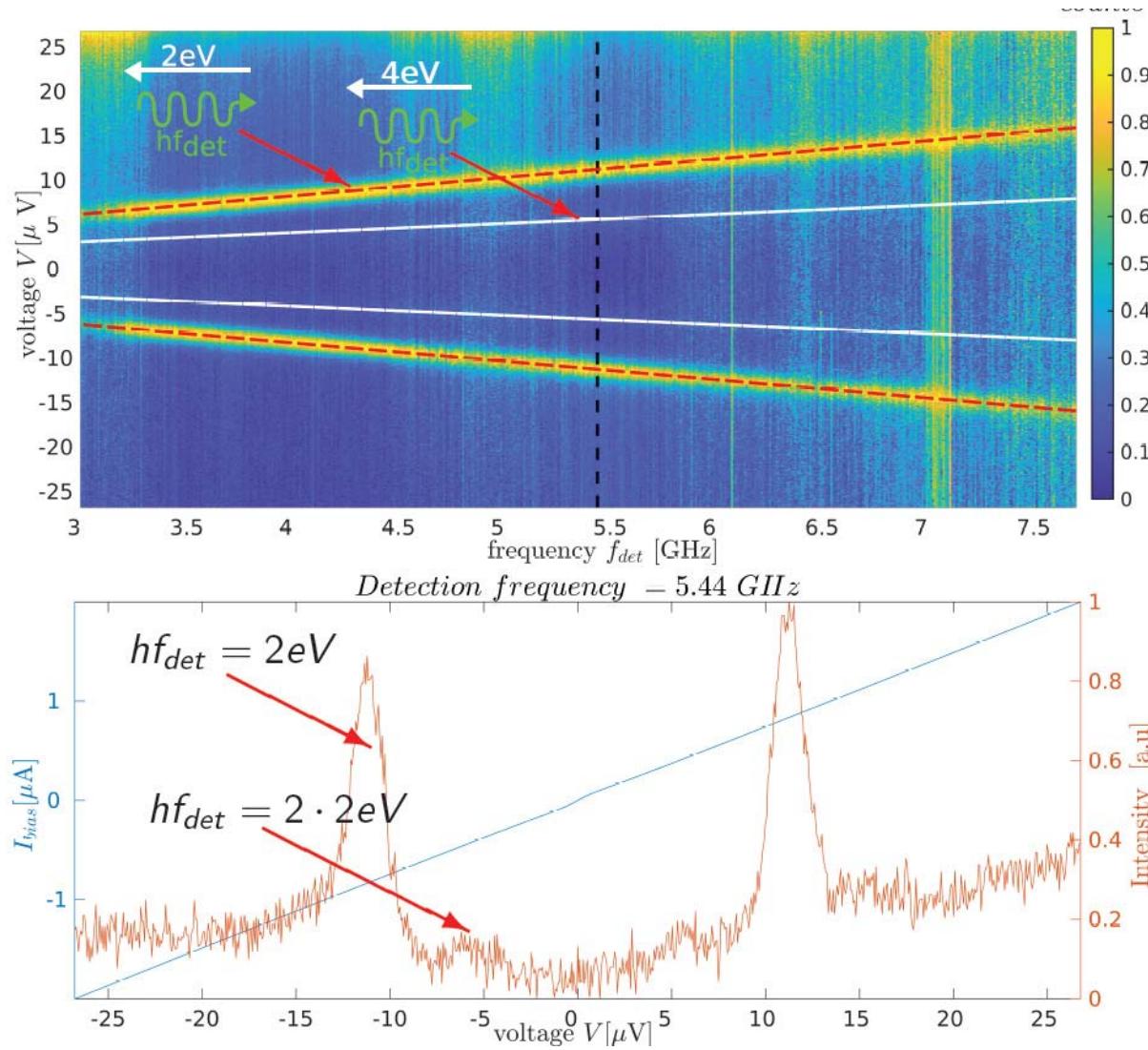
P. Krogstrup et al., Nature Materials **14**, 400 (2015)

- Deposit on bottom gate structure
- Partially remove Al by wet-etching
- Sputtered NbTiN contacts

Al-InAs-NW-Al as reference



... and here the measurement



The **main peak** corresponds to the usual **2π Josephson radiation**, the normal AC Josephson effect:
i.e. $hf = 2eV$.

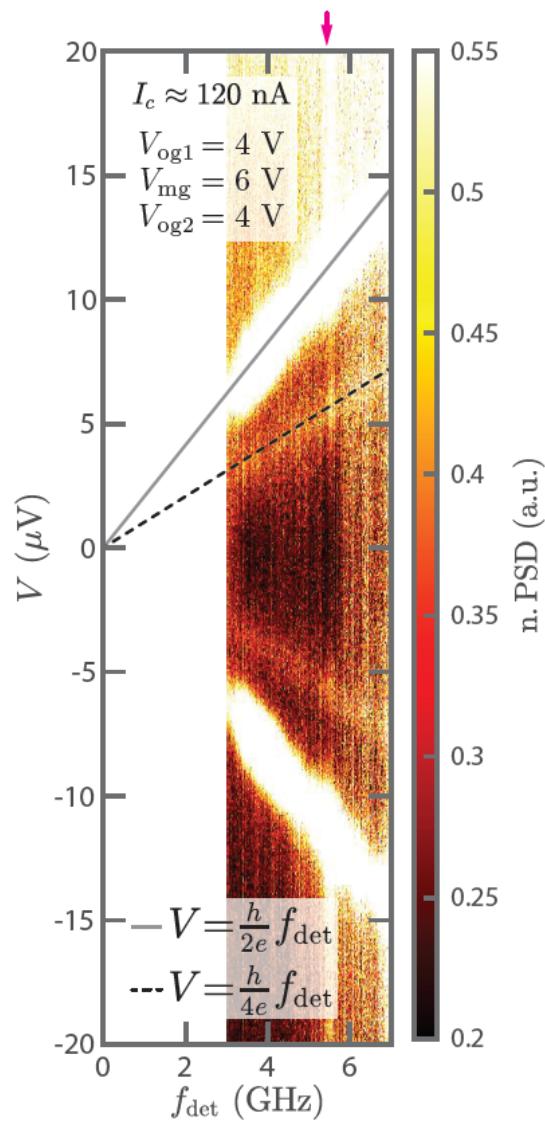
The much weaker peak for the same frequency at half the voltage is a second order process. Here, the inelastic tunneling is accomplished by the transfer of **two Cooper-pairs per photon**.

A 4π signal would show up with twice the slope as compared to the conventional 2π Josephson radiation

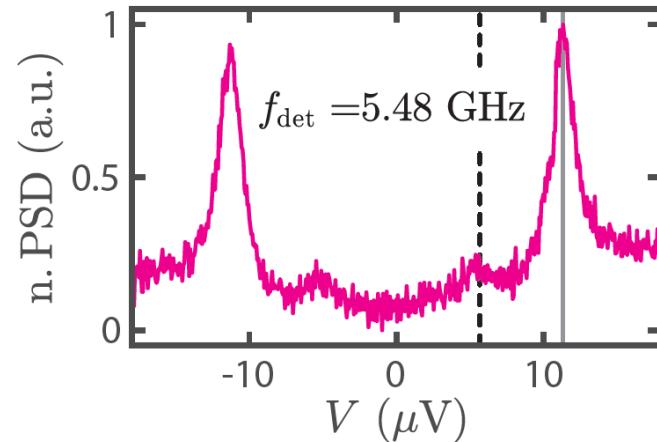
Al-InAs-NW-Al as reference



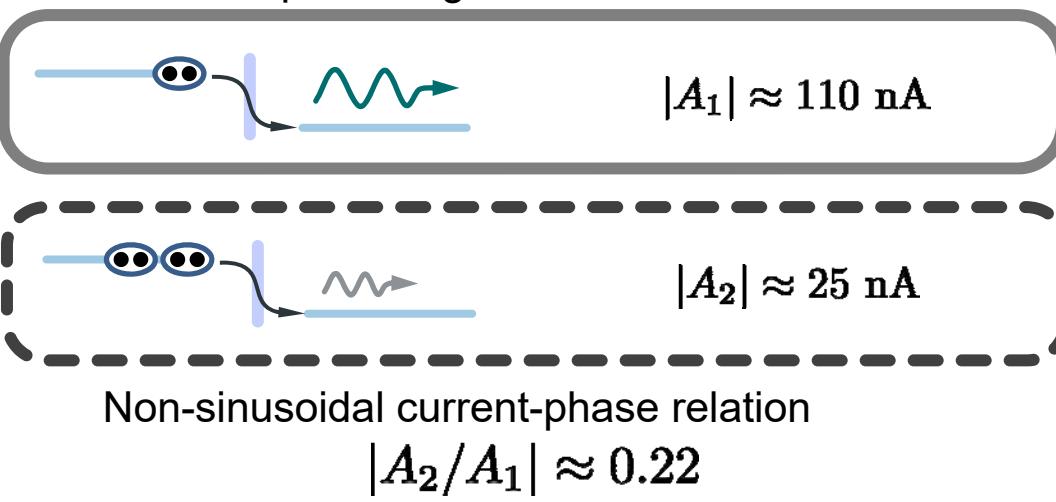
... and here the measurement



Fixed detection frequency

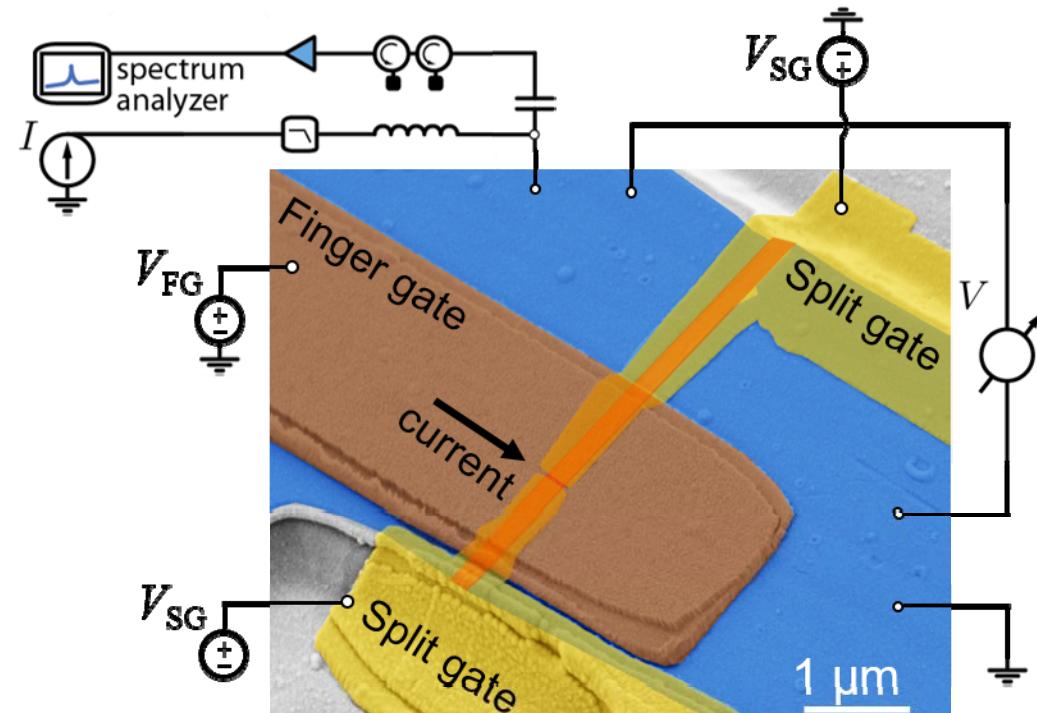
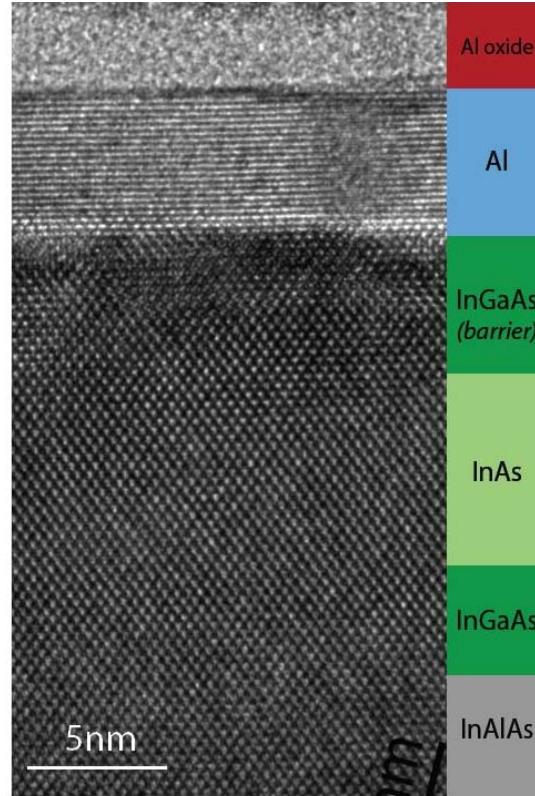


From the peak height ratio:



by Roy Haller et al. (PhD thesis) in collaboration with J. Nygard et al (Copenhagen)

Further material platform: InAs QW



J. Shabani *et al.*, Phys. Rev. B **93**, 155402 (2016)

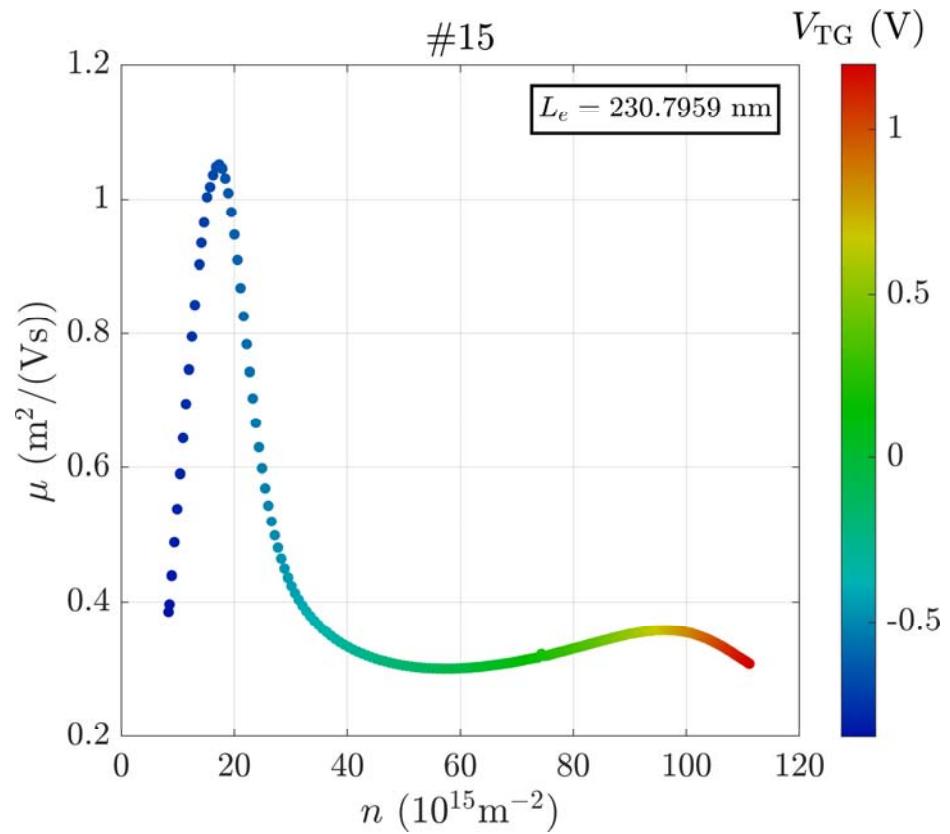
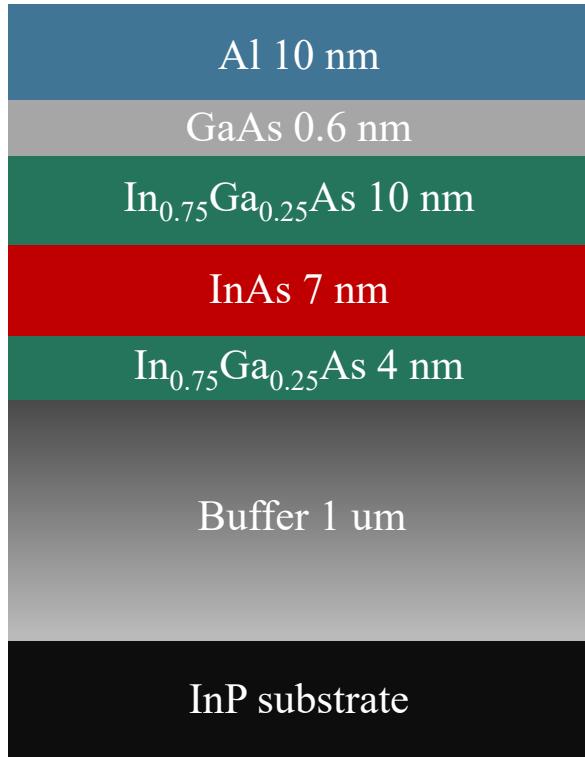
- Epi-Al on InAs 2DEG
- S-QPC is defined and tuned by two split gates and on finger gate

by Carlo Ciaccia & Libin Wang in collaboration with M. Manfra group (Purdue)& C. Marcus group (Copenhagen)

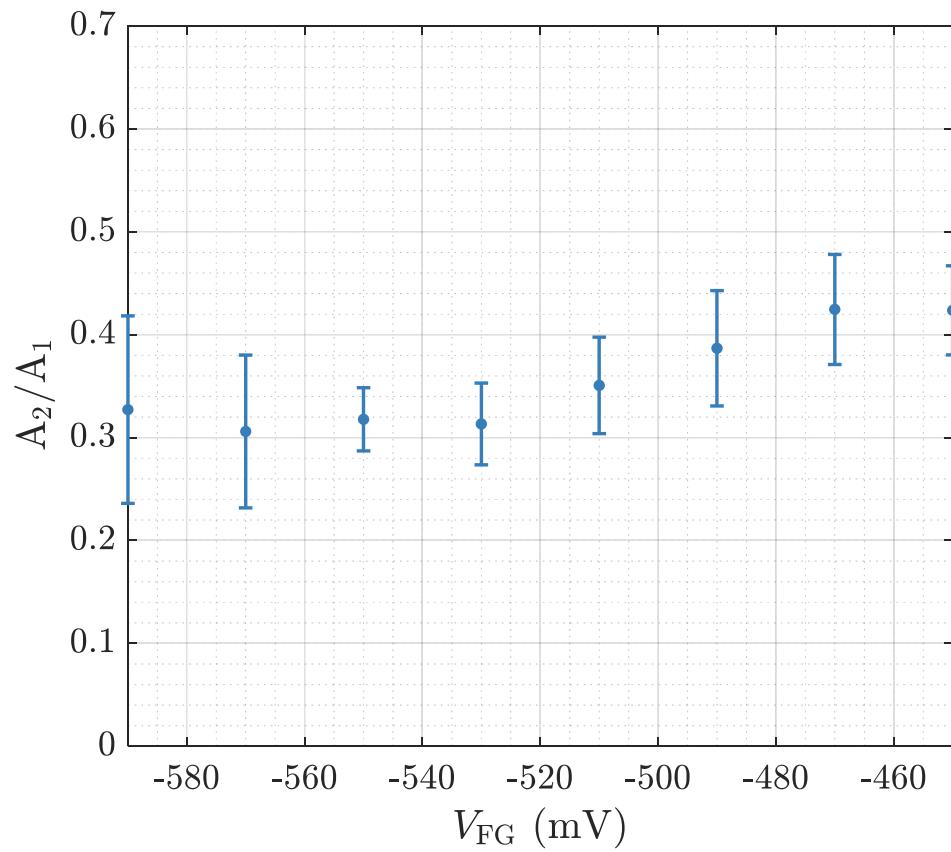
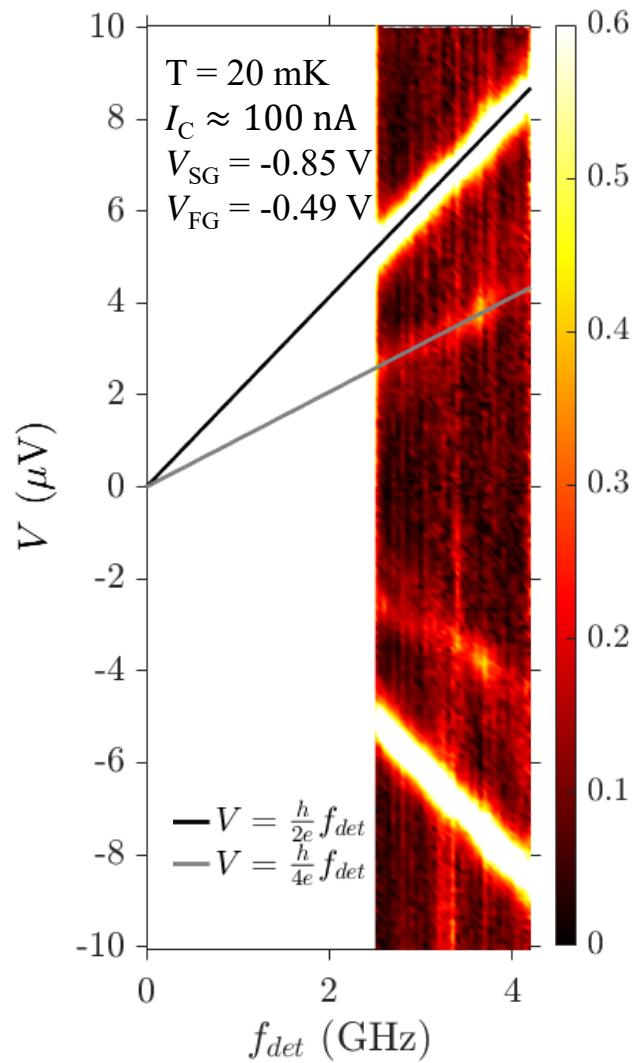
Further material platform: InAs QW



- Wafer no. M-11-11-16
- Sample: C3
- Measurements: Triton, Roy's PCB with 10 Ohm shunt resistor, → 15.7.2021



Further material platform: InAs QW



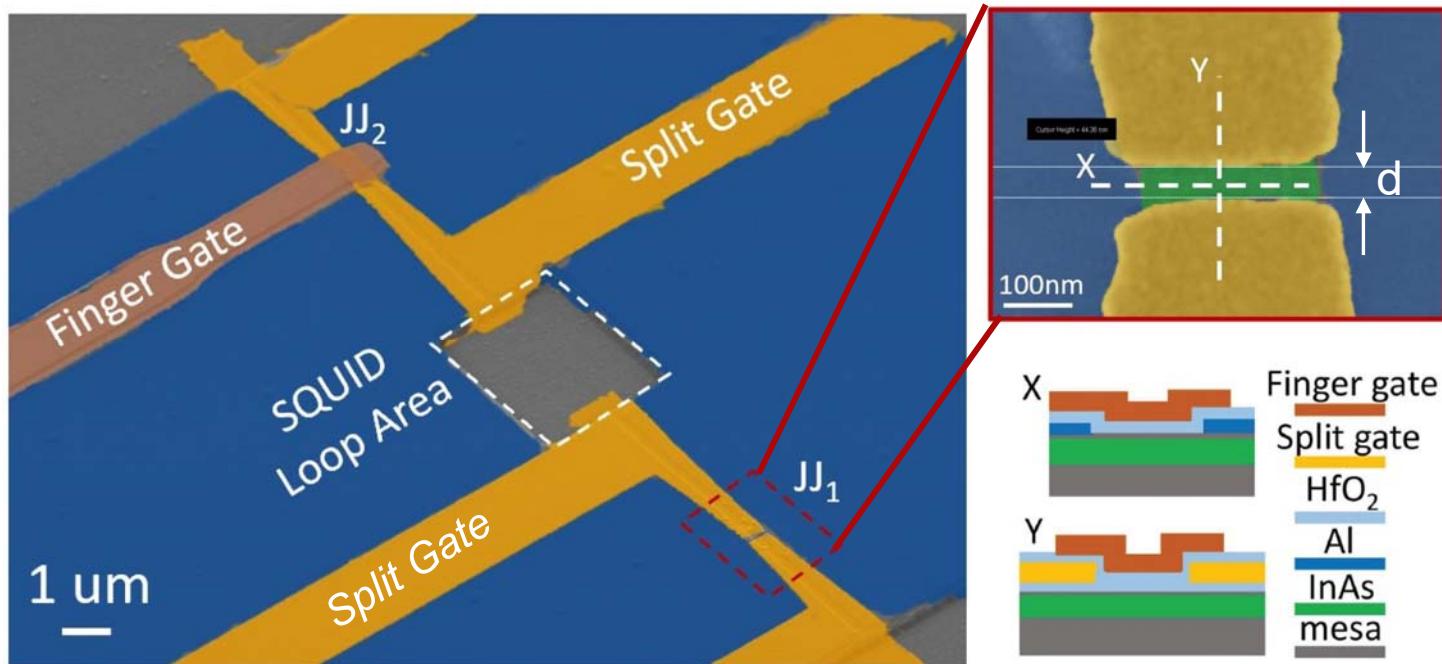
Strongly non-sinusoidal current-phase relation
 $|A_2/A_1| \approx 0.4$

by Carlo Ciaccia & Libin Wang in collaboration with M. Manfra group (Purdue)& C. Marcus group (Copenhagen)

Further material platform: InAs QW



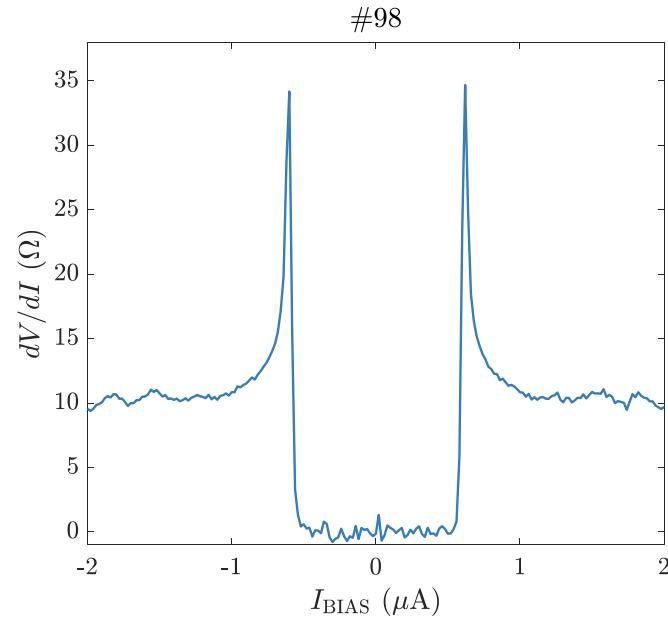
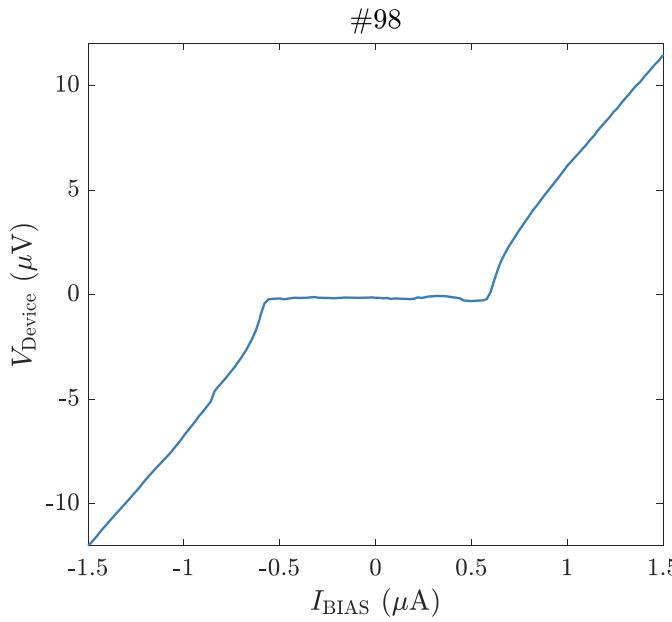
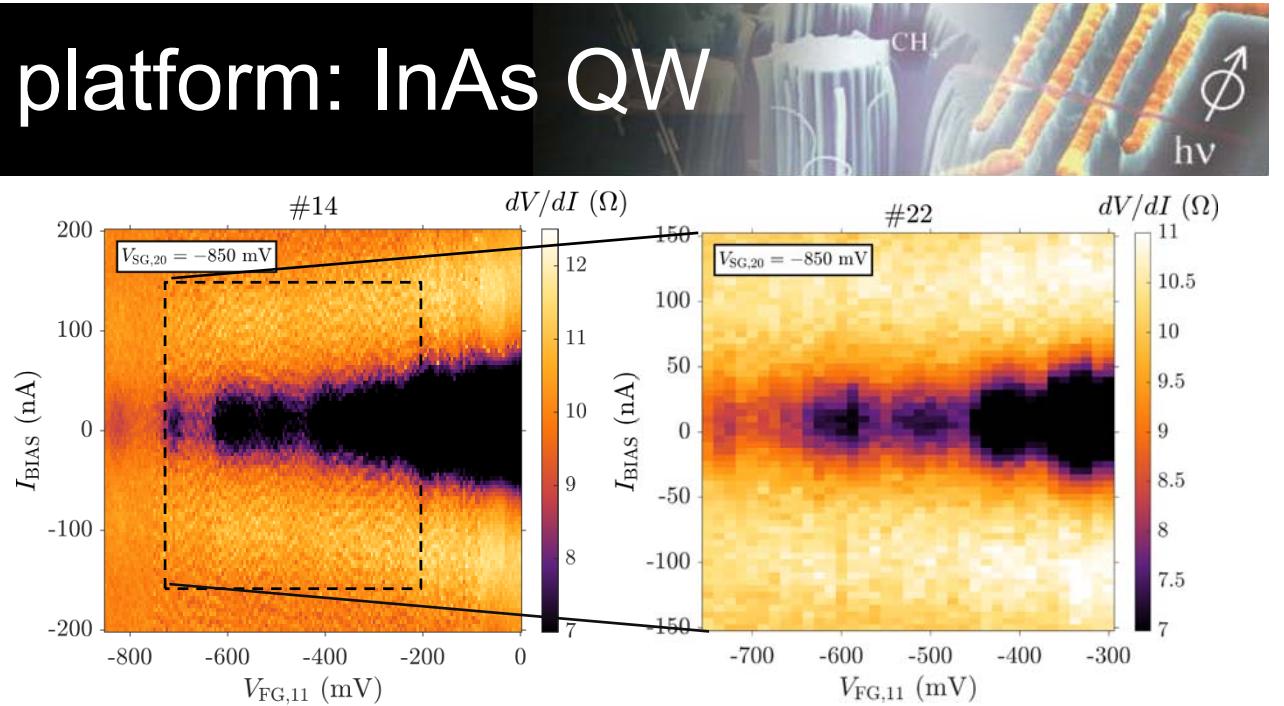
actually, we have fabricated DC-SQUIDs



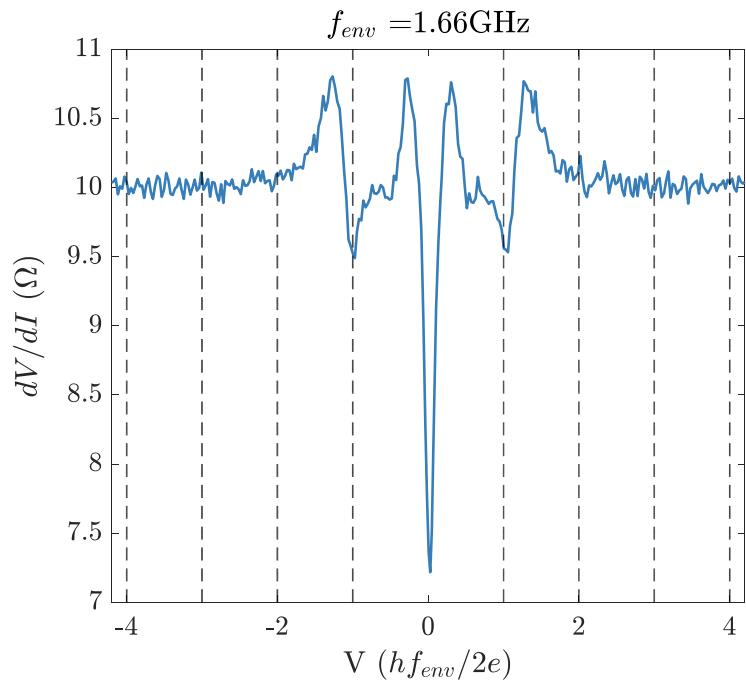
- Junction length ~145 nm, width 4 μ m
- SQUID loop area: 8.9 μ m x 8.9 μ m
- Split gate separation: d = 40 nm (JJ1), 80 nm (JJ2)
- ALD thickness:
 - ✓ First layer: 15 nm
 - ✓ Second layer: 25 nm

Further material platform: InAs QW

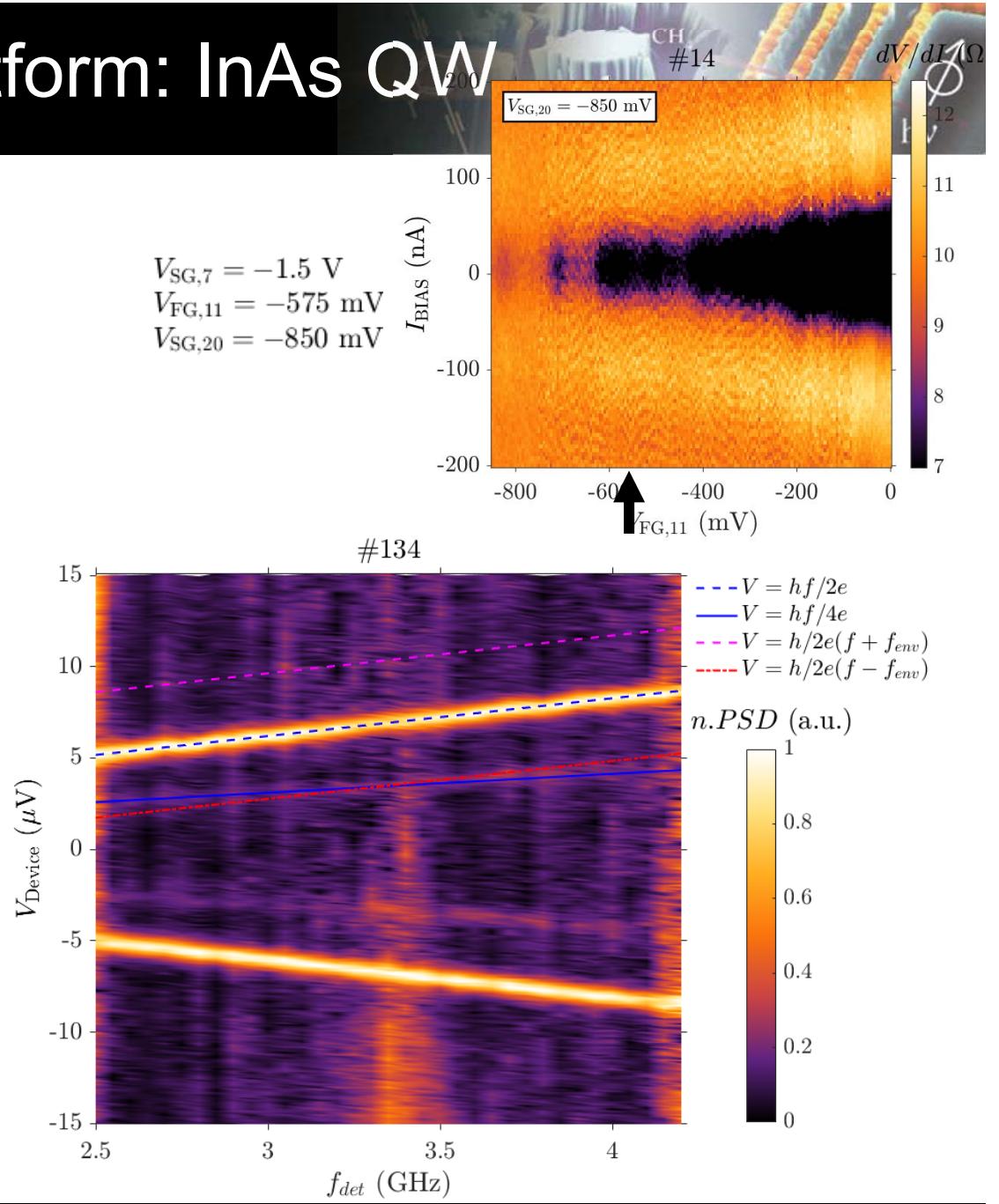
- 24 mK
- JJ1 depleted, $V_{sg}^1 = -1.5V$
- JJ2: $V_{sg}^2 = V_{fg}^2 = 0V$
- No hysteresis
- Two JJs have comparable I_c
(JJ1 not shown here)



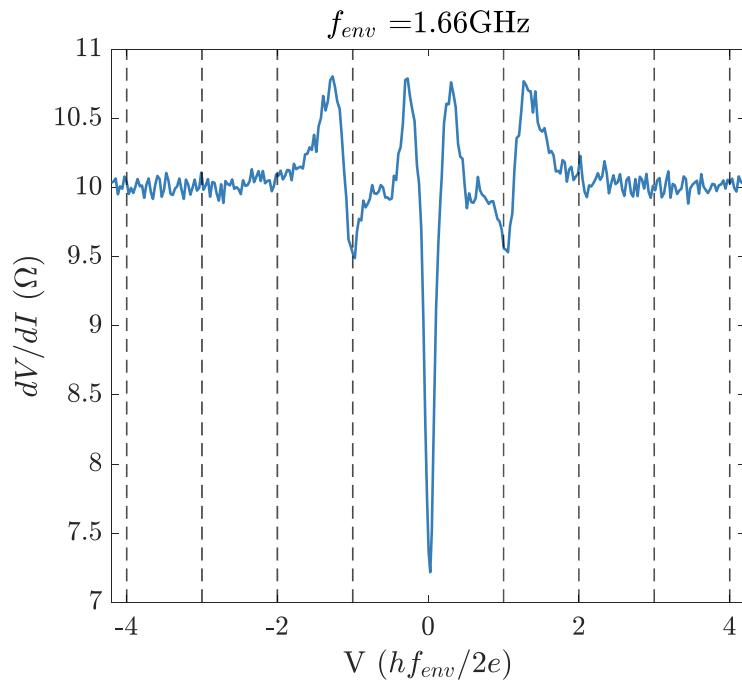
Further material platform: InAs QW



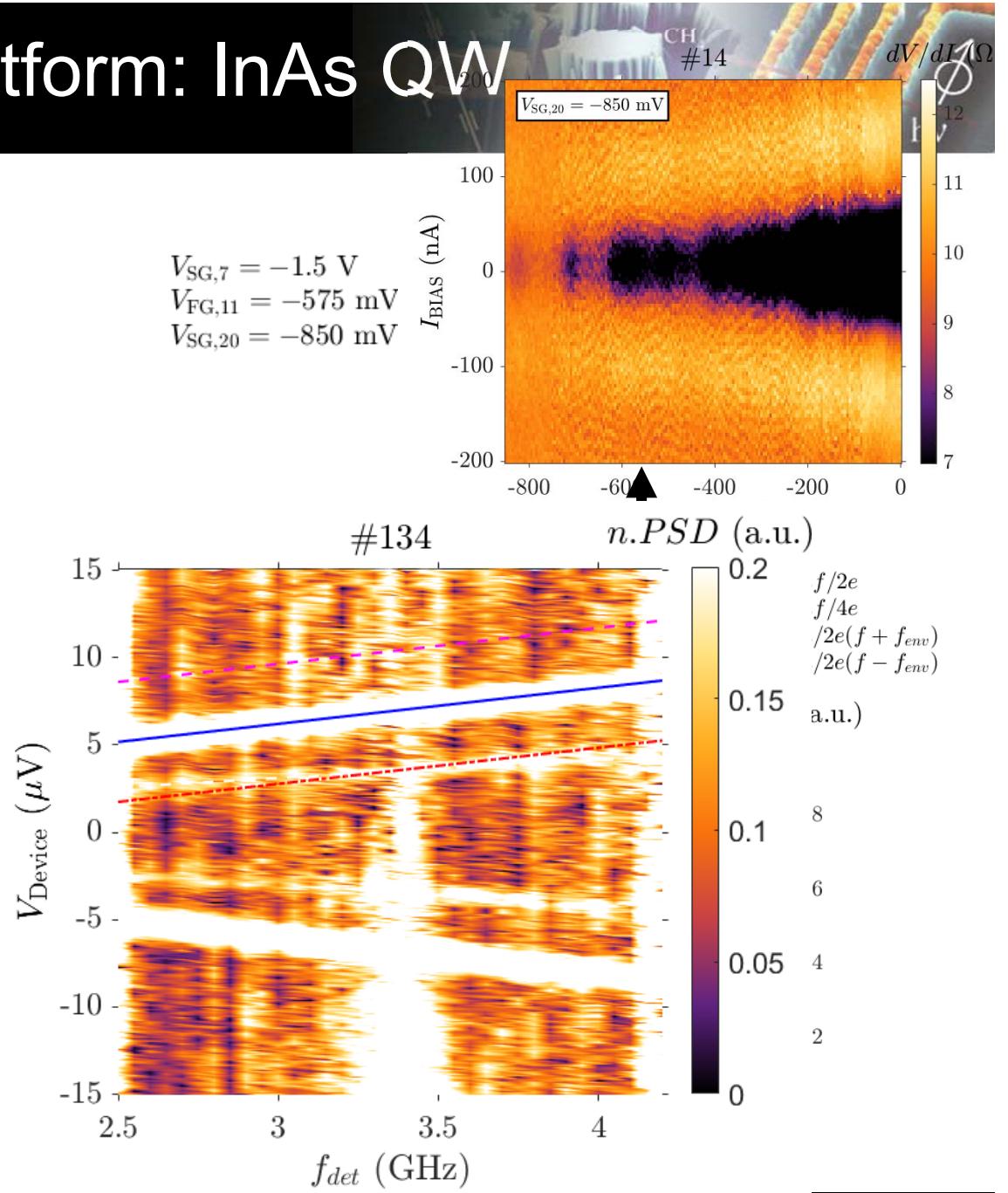
Not very obvious signature of environment coupling



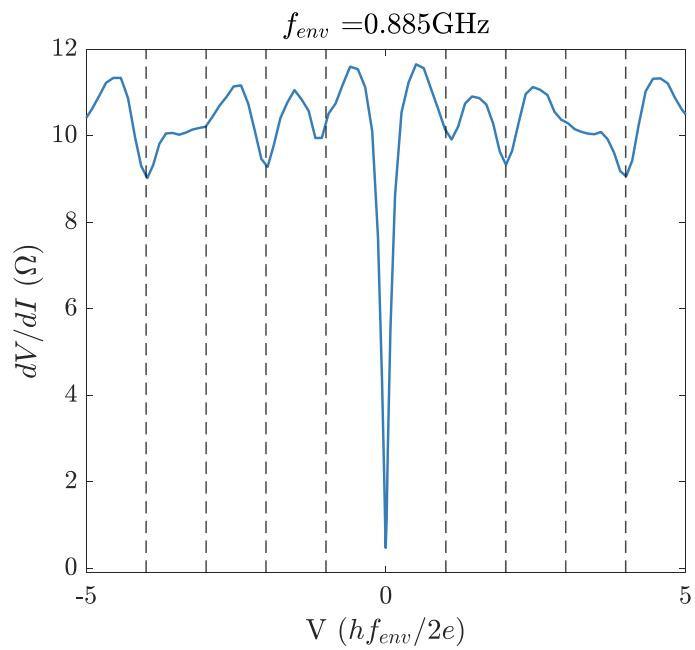
Further material platform: InAs QW



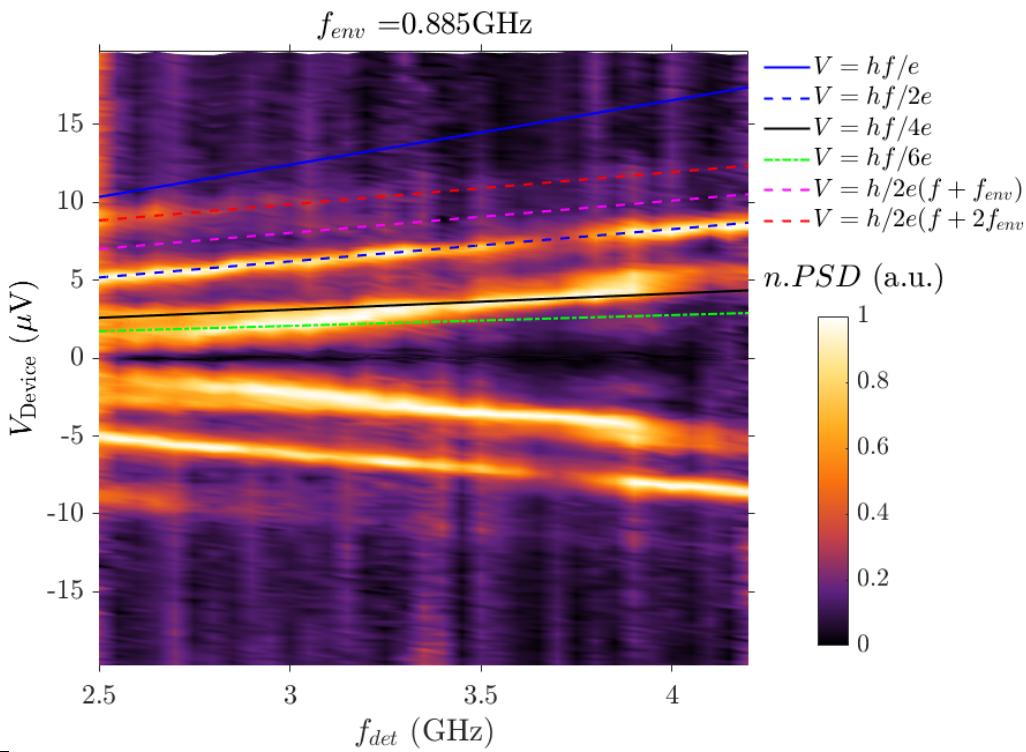
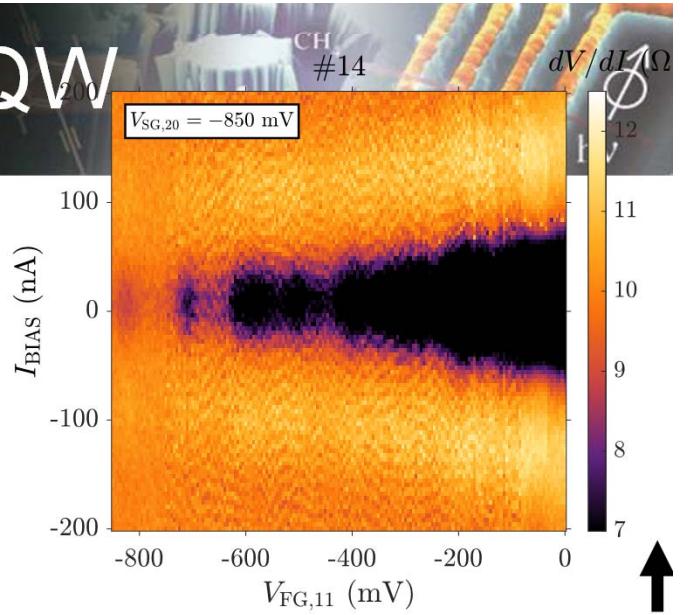
Not very obvious signature of environment coupling



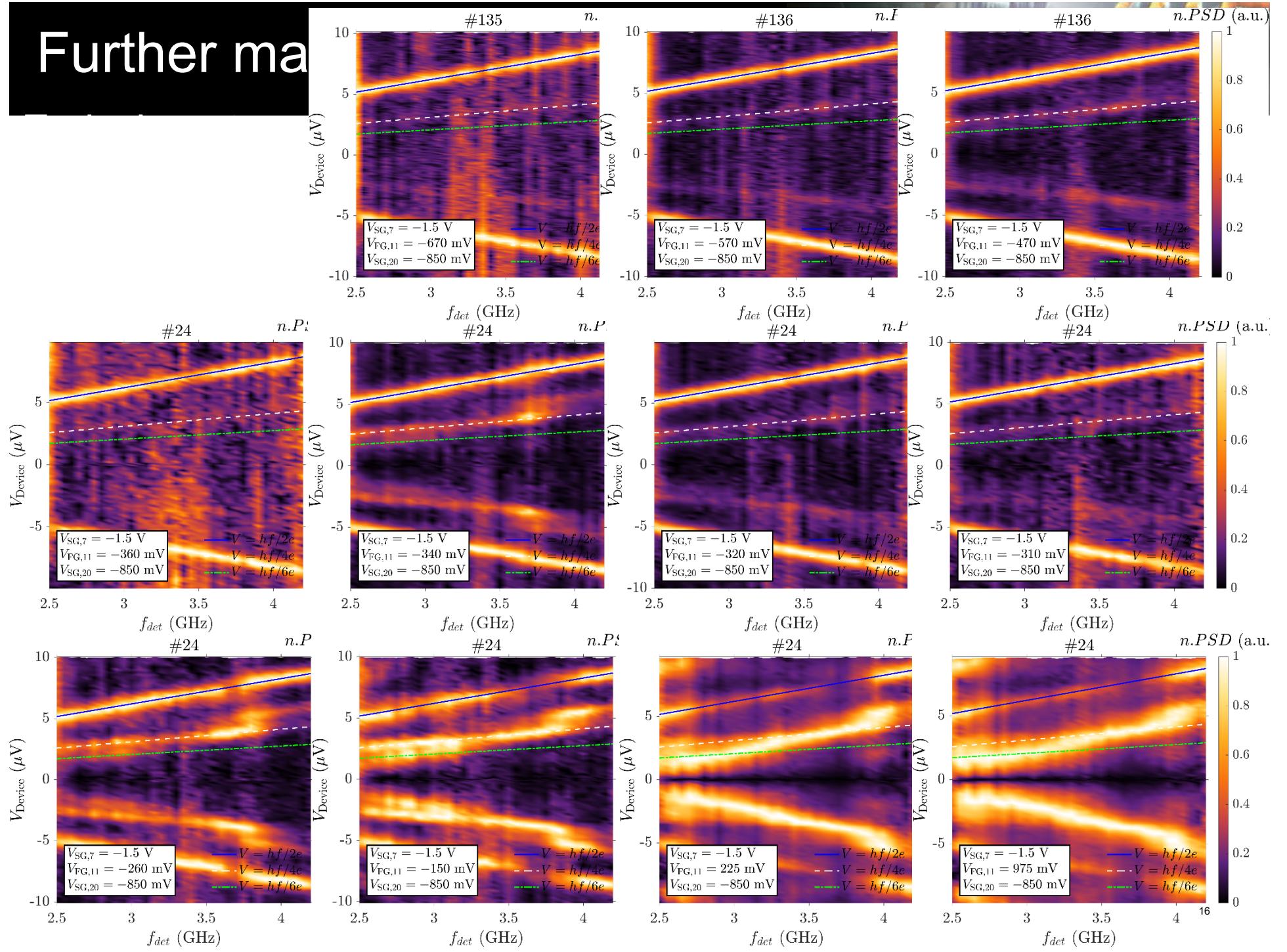
Further material platform: InAs QW



$V_{SG,7} = -1.5 \text{ V}$
 $V_{FG,11} = 200 \text{ mV}$
 $V_{SG,20} = -850 \text{ mV}$



Further ma



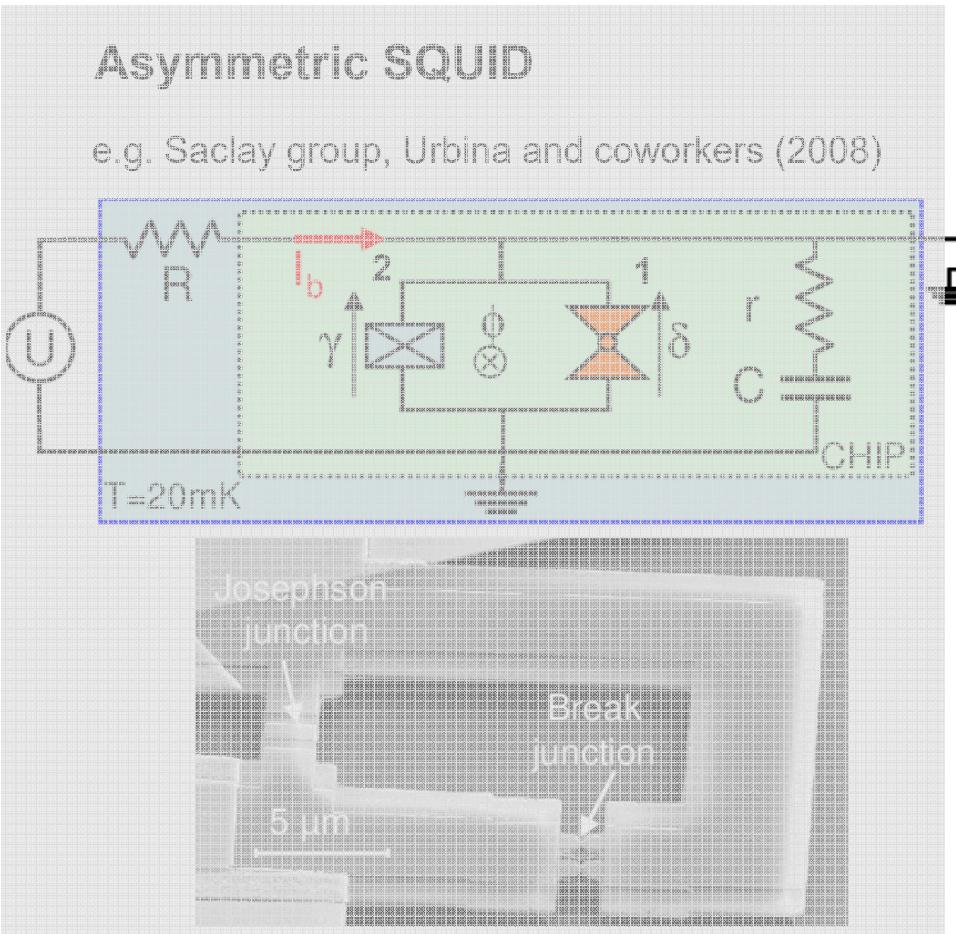
Lecture II

Introduction to CPR by DC-SQUID measurements

Measure the Current-Phase Relation

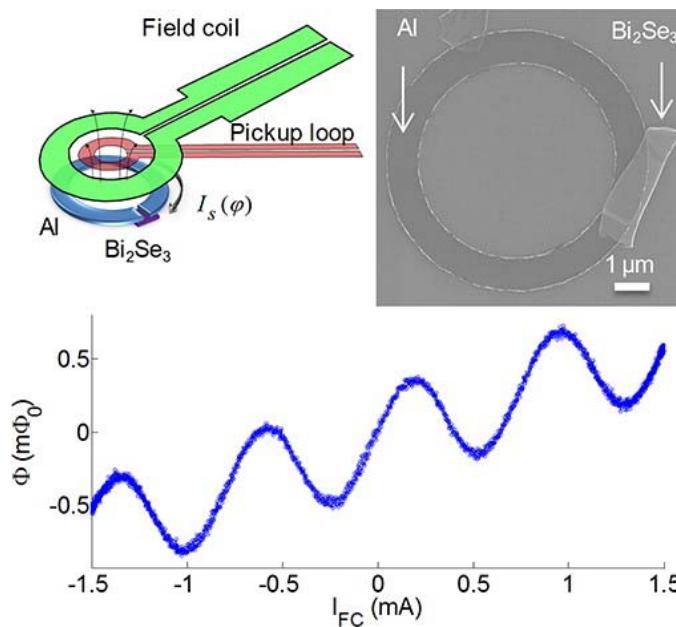
Asymmetric SQUID

e.g. Saclay group, Urbina and coworkers (2008)



Rf susceptibility

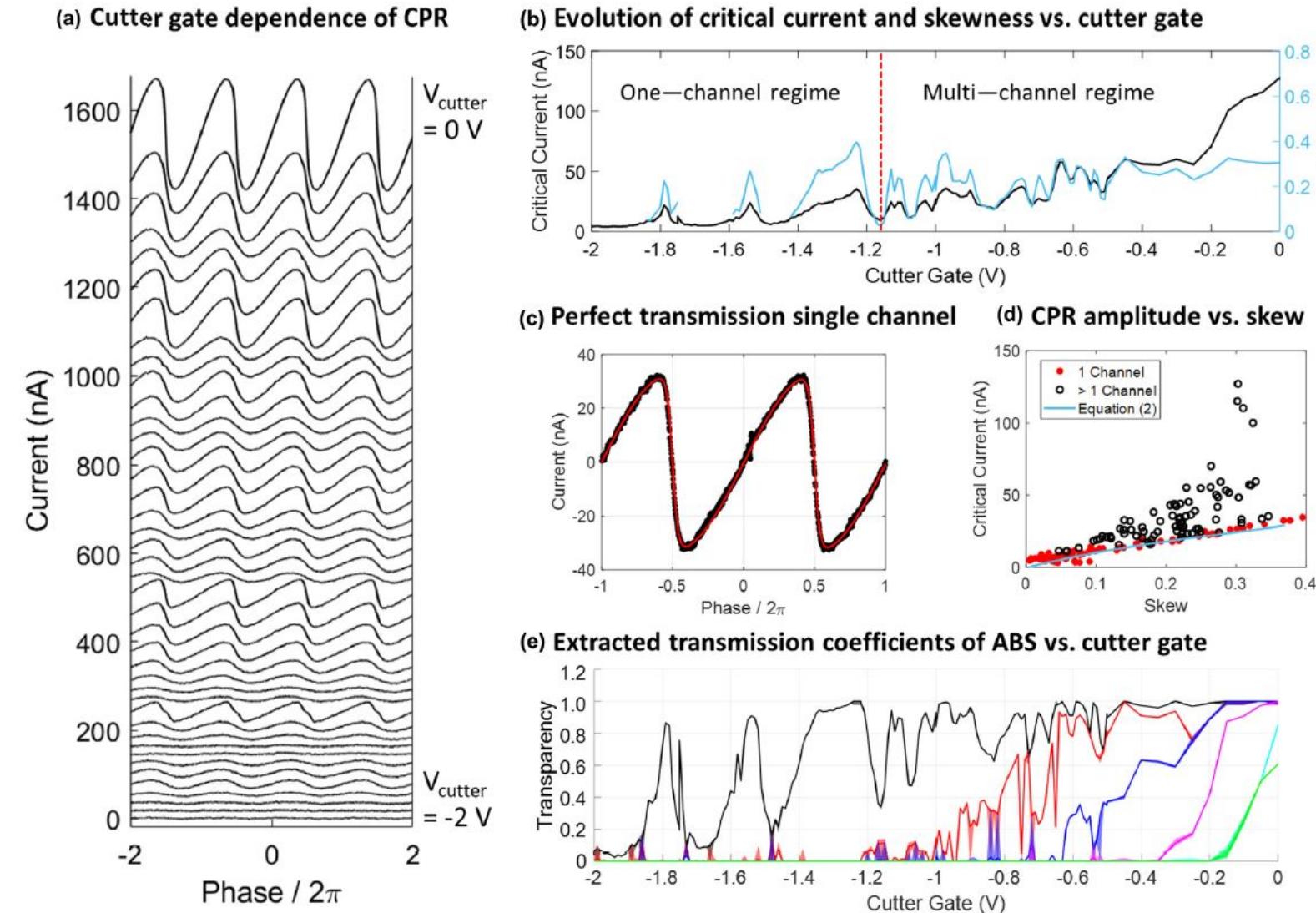
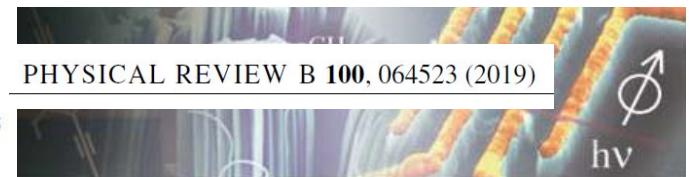
Rf-SQUID: *Nano Lett.* 2013, 13, 3086-3092



Kathryen A. Moler et al.

**Current-phase relations of InAs nanowire Josephson junctions:
From interacting to multimode regimes**

Sean Hart,^{1,2,*} Zheng Cui,^{1,2,3,*} Gerbold Ménard,⁴ Mingtang Deng,⁴ Andrey E. Antipov,⁵ Roman M. Lutchyn,⁵ Peter Krogstrup,^{4,6} Charles M. Marcus,⁴ and Kathryn A. Moler^{1,2,3}

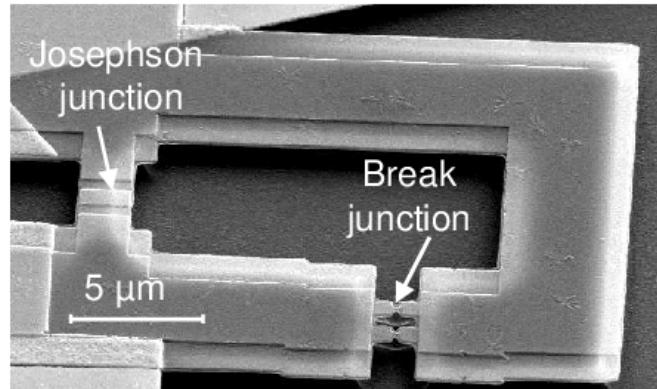
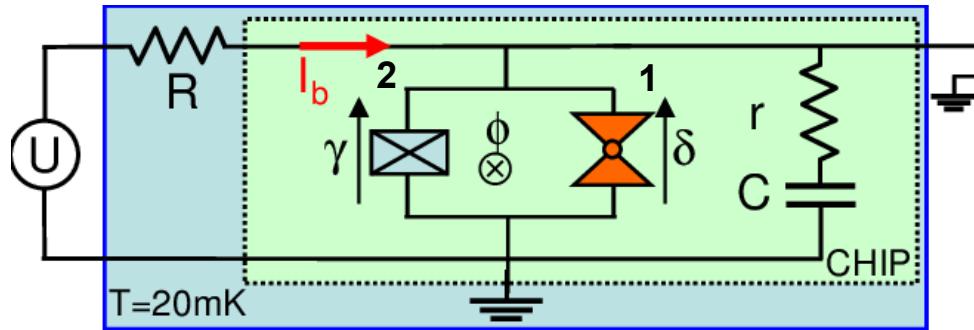


Measure the Current-Phase Relation



Asymmetric SQUID

e.g. Saclay group, Urbina and coworkers (2008)



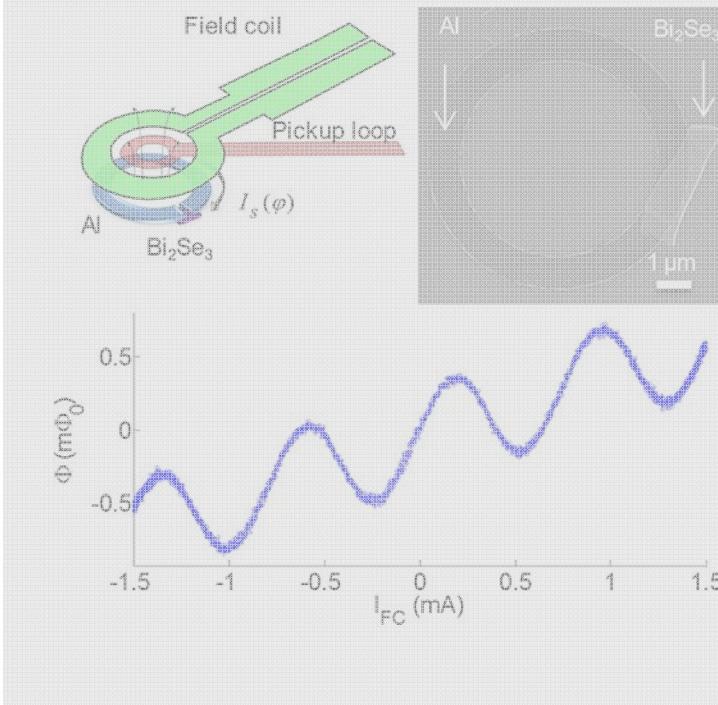
$$I_s = I_1(\delta) + I_2(\gamma)$$

$$\delta - \gamma = 2\pi\phi/\phi_0 \quad \text{fluxoid relation}$$

$$I_c(\phi) = \max_{\delta, \gamma} |I_1(\delta) + I_2(\gamma)| \rightarrow I_c(\phi) = I_{c2} + I_1(2\pi\phi/\phi_0 + \gamma_{2c}) \quad I_{c2} = I_2(\gamma_{2c})$$

Rf susceptibility

Rf-SQUID: *Nano Lett.* 2013, 13, 3086-3092



CPR by asymmetric SQUID

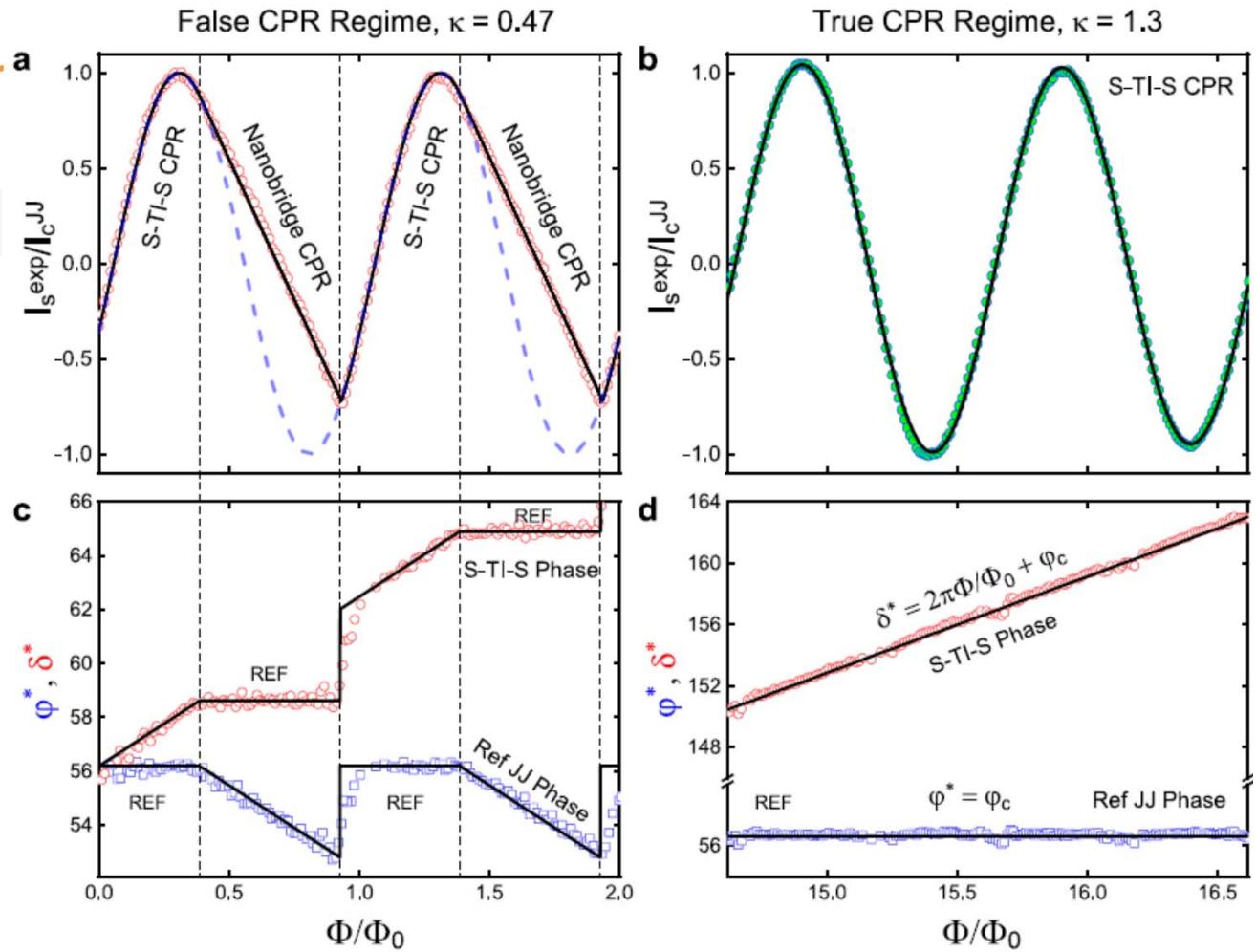
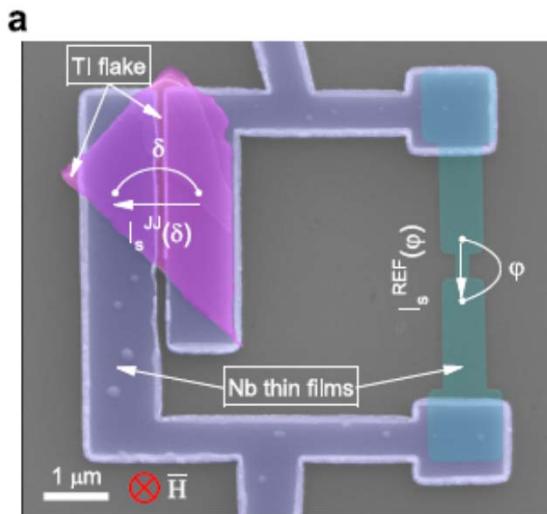


Limitations of the Current–Phase Relation Measurements by an Asymmetric dc-SQUID

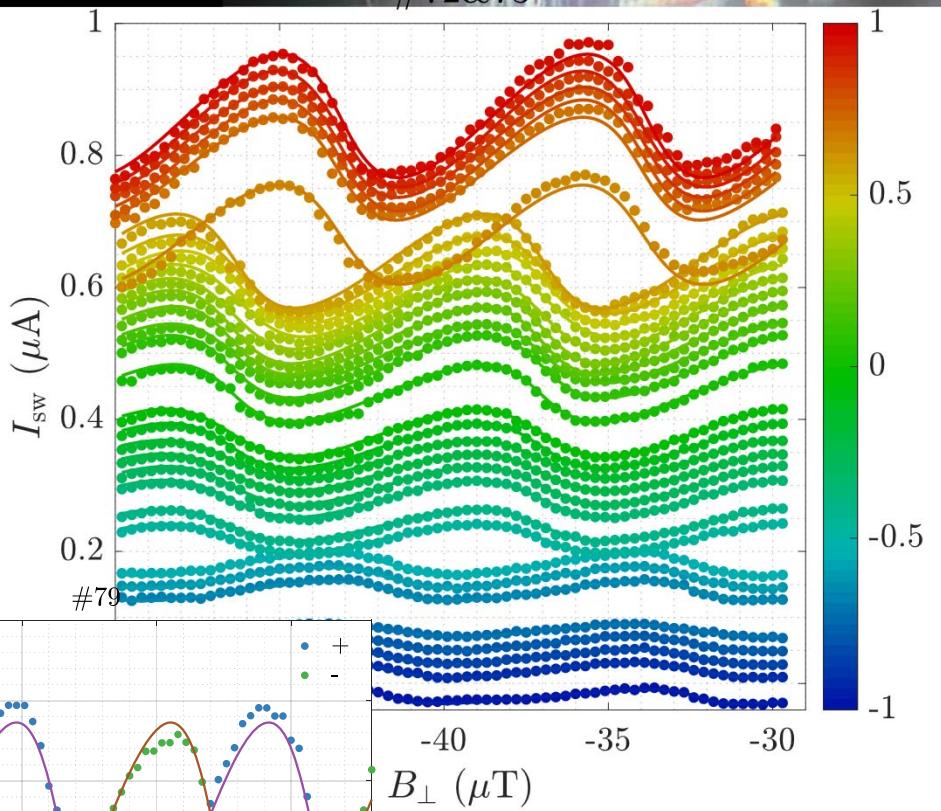
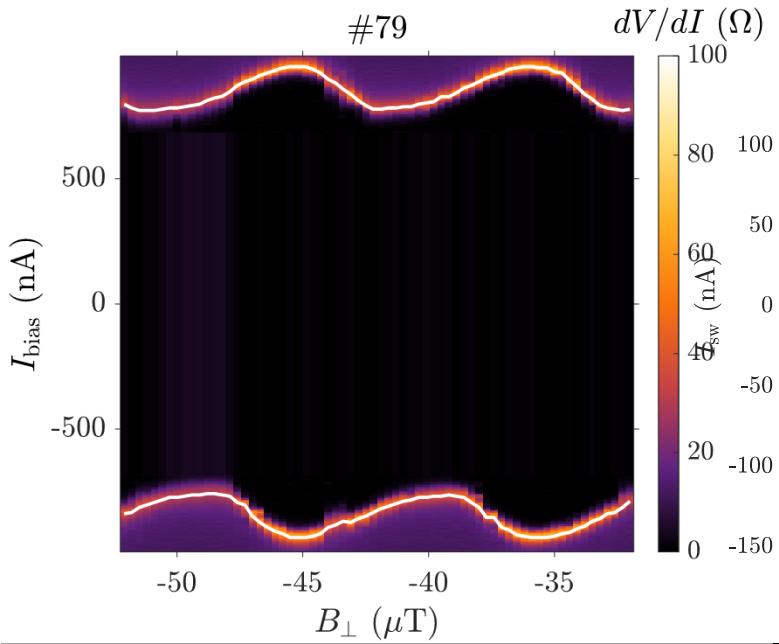
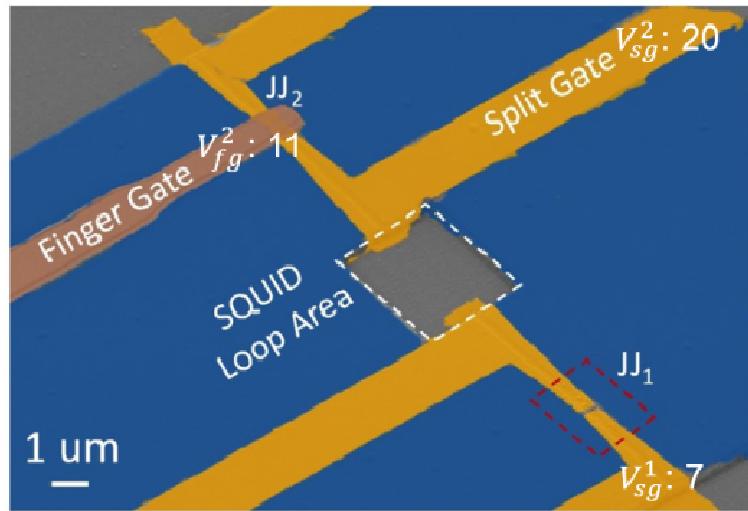
Ian Babich,* Andrei Kudriashov, Denis Baranov, and Vasily S. Stolyarov

Cite This: *Nano Lett.* 2023, 23, 6713–6719

study JJ from 3D TI $\text{Bi}_2\text{Te}_2\text{Se}$



CPR by asymmetric SQUID



Yes, there is some **skewness** and hence, we have to expect **higher harmonics in AC radiation** (which we did). But, it is **not** straightforward to extract the skewness from the relative intensities of the harmonics, see:

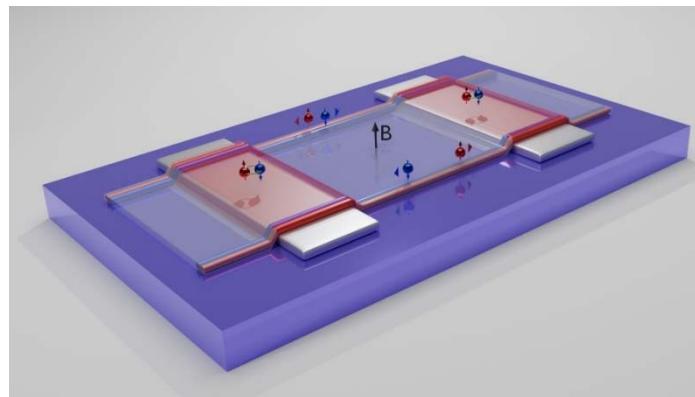
R. Haller, M. Osterwalder, G. Fülöp, J. Ridderbos, M. Jung, and C. Schönenberger,
Phys. Rev. B **108**, 094514 (2023)

Lecture II

A study with the material WTe₂

An experimental search for topology in WTe_2

Lecture II: Focus more on CPR using the asymmetric SQUID approach



Martin Endres, Artem Kononov, Christian Schönenberger
Quantum- and Nanoelectronics group

Team



Samples and measurements:

A. Kononov, M. Endres, G. Abulizi and C. Schonenberger

Department of Physics, University of Basel



WTe₂ growth:

H.S. Arachchige, K. Qu, J. Yan, D. G. Mandrus

Materials Science and Engineering, The University of Tennessee



Funding:



TEM-STEM EDX studies:

Marcus Wyss

Nanolmaging Lab @ Swiss Nanoscience Institute, Univ. of Basel



hBN growth:

Kenji Watanabe, Takashi Taniguchi

Advanced Materials Laboratory, National Institute for Materials Science



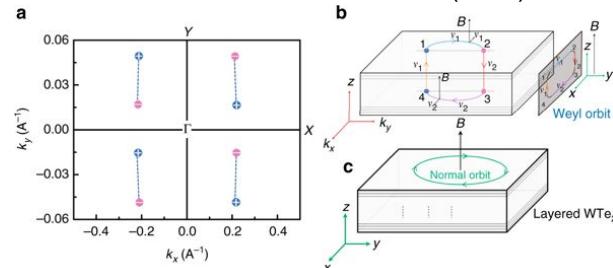
WTe₂ intriguing properties



Nontrivial topology

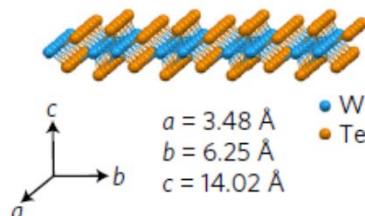
- ✓ Bulk crystal is type II Weyl semimetal

P. Li, Y. Wen, X. He, Nature Comm. 8, 2150 (2017)



- ✓ Monolayer is 2D topological Insulator

Z. Fei, T. Palomaki, S. Wu et al., Nature Physics 13, 677 (2017)



❑ Higher-Order Topological insulator

Z. Wang, B.J. Wieder, J. Li, B. Yan, B.A. Bernevig, arXiv:1806.11116
A. Kononov et al., Nano Lett. 20, 6, 4228 (2020)

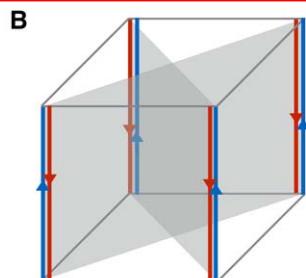


Image: F. Schindler et al., Science Advances 4, no. 6, eaat0346 (2018)

Superconductivity

- ✓ Under high pressure is superconducting

X.-Ch. Pan, X. Chen, H. Liu, Nature Comm. 6, 7805 (2015)

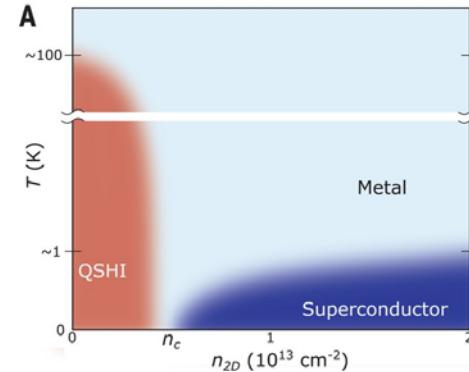
- ✓ Superconducting when doped

T. Asaba, Y. Wang, G. Li et al., Scientific Rep. 8, 6520 (2018)

- ✓ Monolayer is tunable with gate into superconducting state

E. Sajadi et al., Science 362, p. 922 (2018)

V. Fatemi et al., Science 362, p. 926 (2018)



❑ Superconductivity at the interface with Pd

A. Kononov, M. Endres et al., Journal of Applied Physics 129, 113903 (2021)

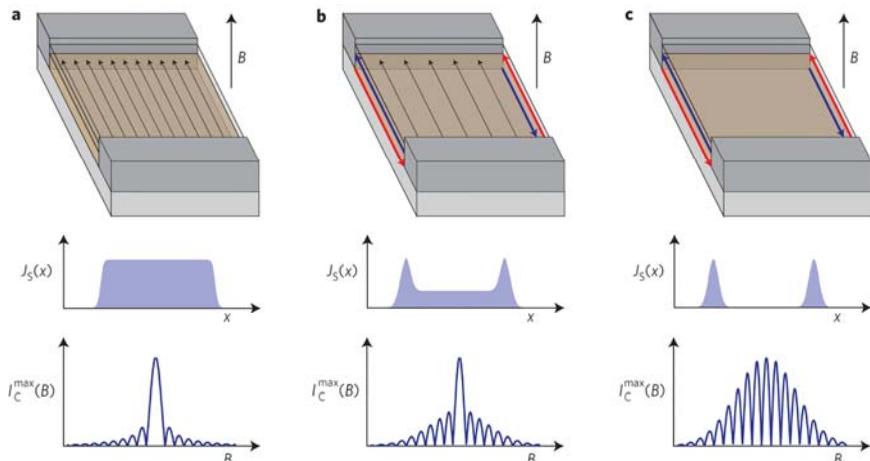
Goal: search for **edge currents** in transport or edge DOS in STM

Josephson junctions in TI materials

Use superconducting interference in wide Josephson junctions to probe the current distribution

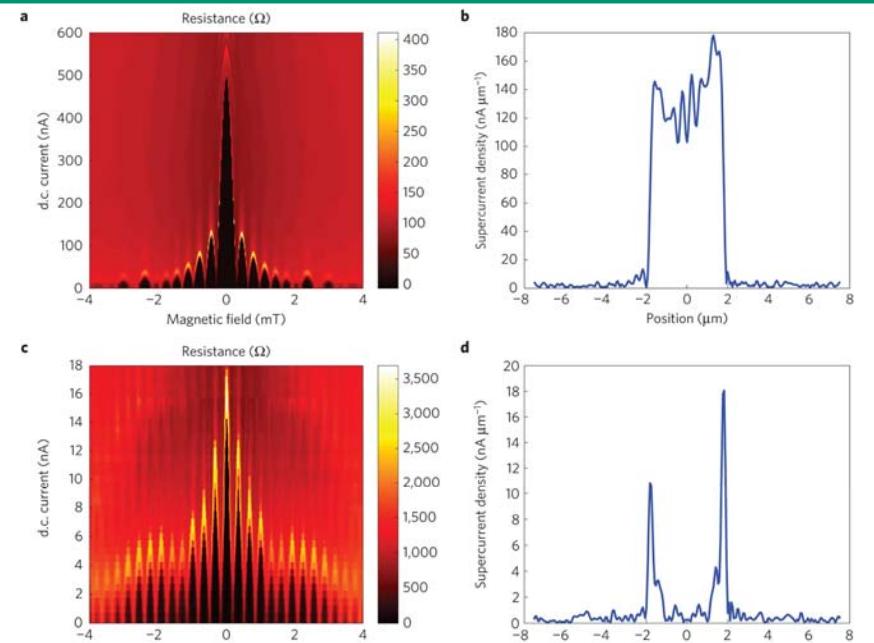
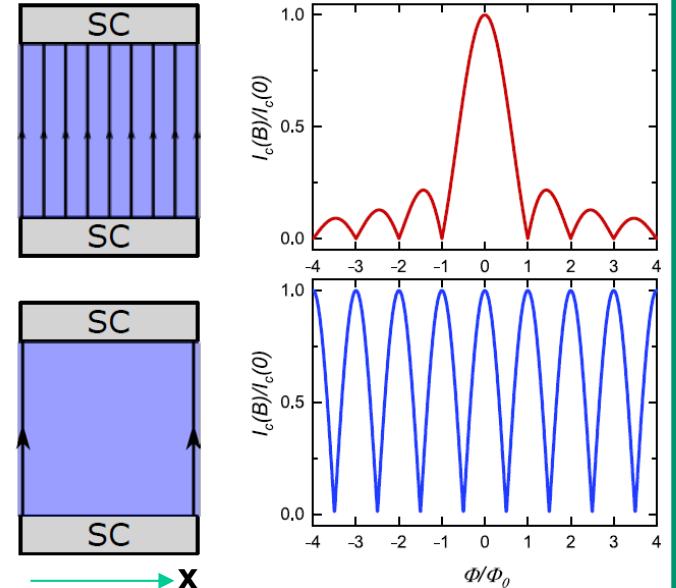
Example:

Induced superconductivity in the quantum spin Hall edge
Yacoby and Molenkamp groups: Nature Physics 10, 238 (2014)



To measure the **current distribution** in plane one makes $B \otimes$ use of quantum interference induced by the electromagnetic gauge field, related to the mag. field B .

The acquired phase is given by the **flux $\Phi(x)$** divided by the flux quantum for a Cooper pair.

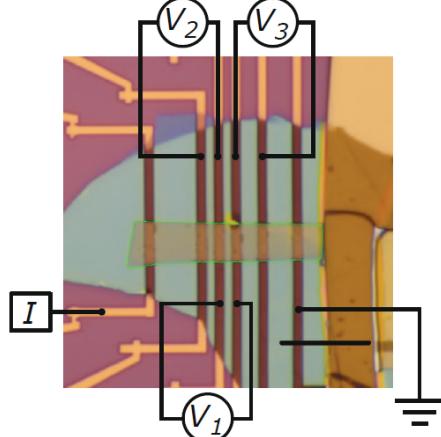


Weyl SM junction: WTe₂

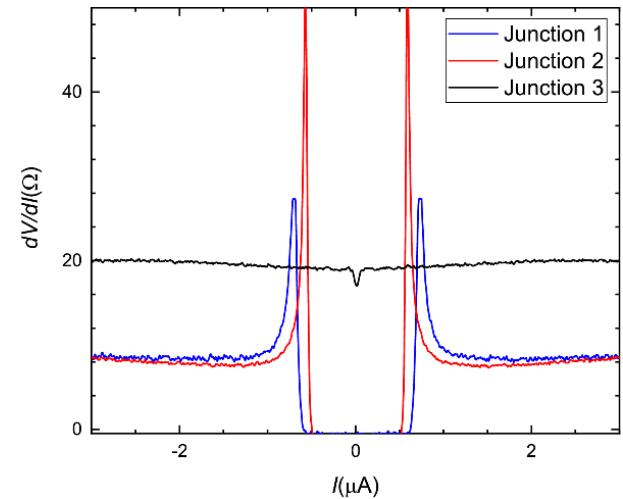
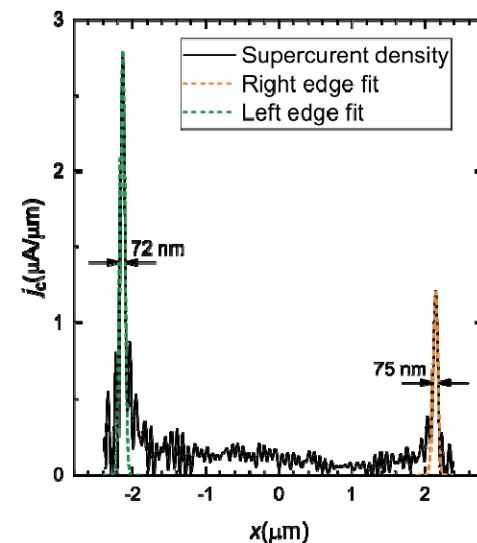
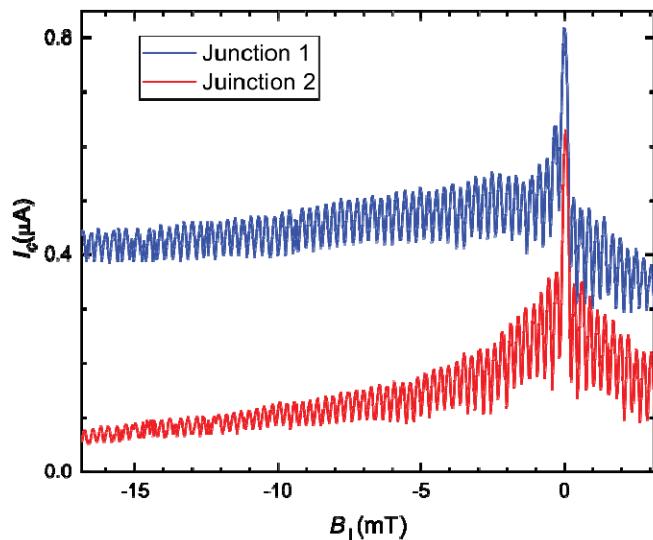


WTe₂ predicted to be higher-order topological insulator

Pd induced superconductivity



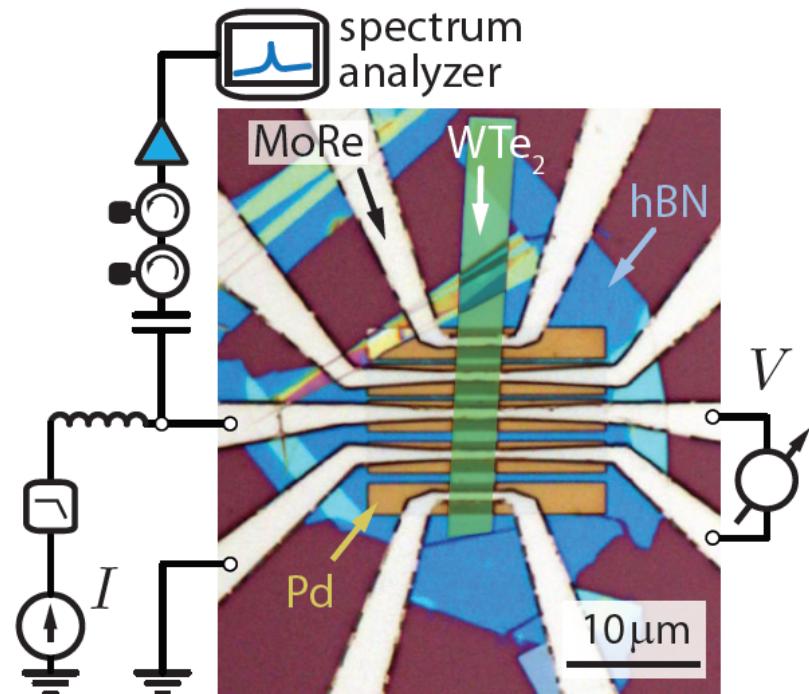
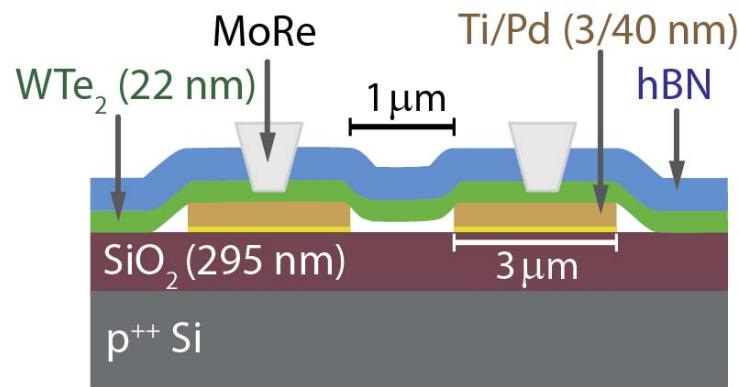
Robust **SQUID-like oscillations** of $I_c(B_\perp)$ highlight presence of extremely narrow edge states



Junctions 1 & 2: $L = 1 \mu\text{m}$
Junction 3: $L = 2 \mu\text{m}$

experiments:
Kononov *et al.*, Nano Letters **20**, 4228 (2020)
Y.-B. Choi *et al.*, Nat. Mater. **19**, 974 (2020)
C. Huang *et al.*, Nat. Sc. Rev. **7**, 1468 (2020)

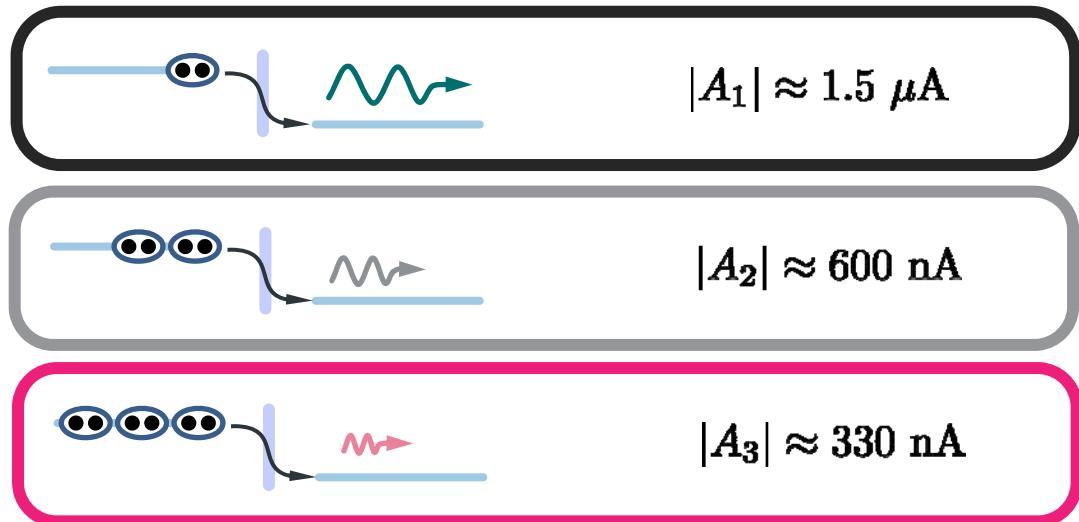
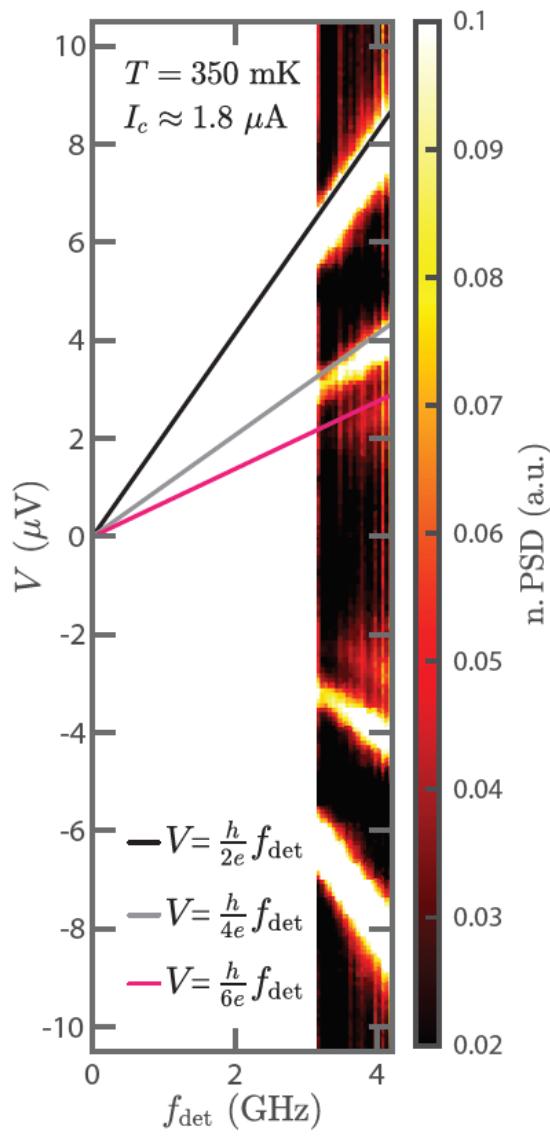
Weyl SM junction: WTe₂



- Pd induced superconductivity Kononov *et al.*, Nano letters **20**, 4228 (2020)
- hBN protection layer
- MoRe side contacts Indolese *et al.*, Nano letters **20**, 7129 (2020)

by Artem Kononov *et al.* in collaboration with D. Mandrus group (Univ. of Tennessee)

Weyl SM junction: WTe₂



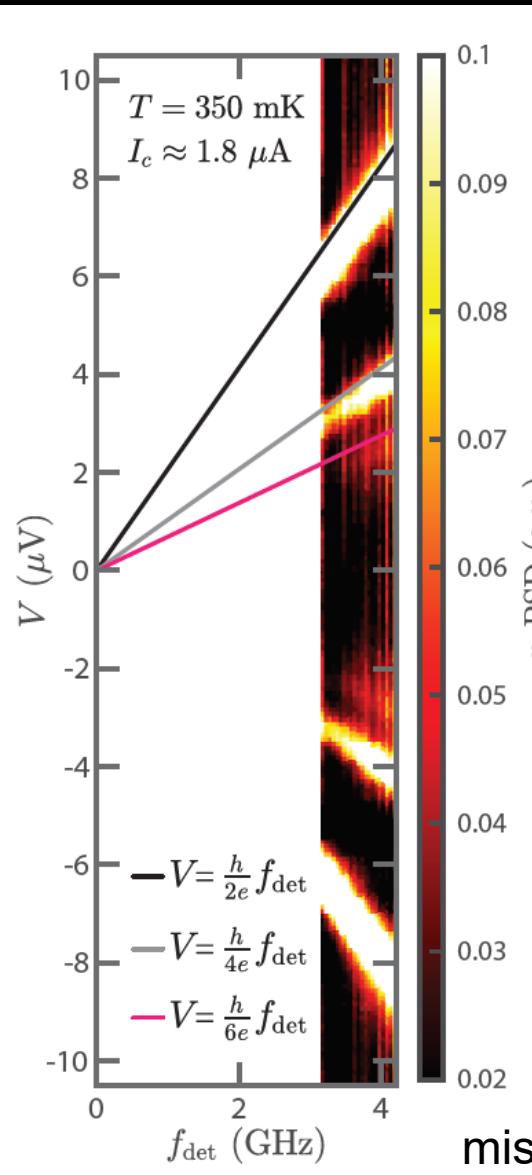
Strongly non-sinusoidal current-phase relation

$$|A_2/A_1| \approx 0.4 \quad |A_3/A_1| \approx 0.22$$

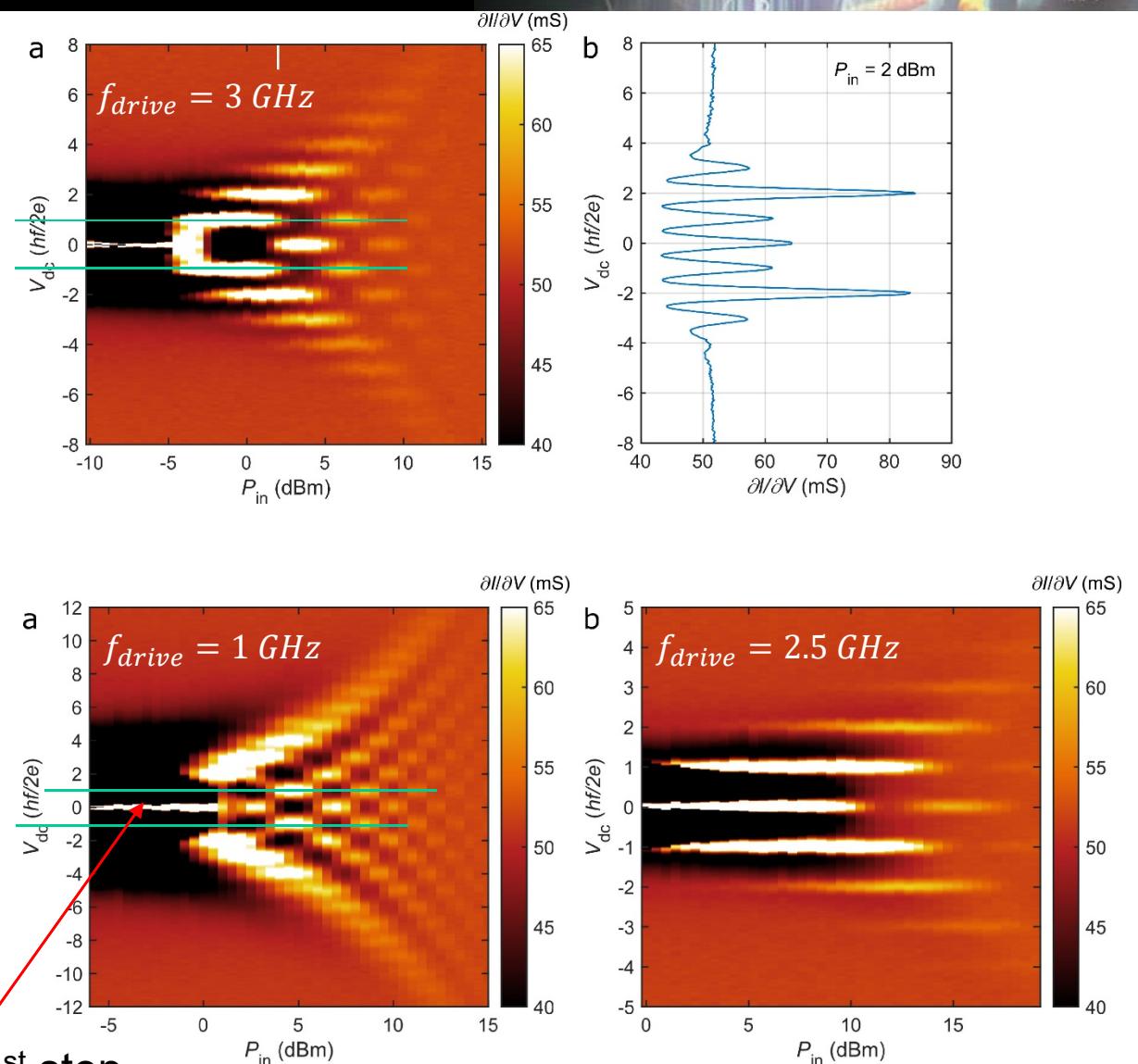
(close to the ballistic limit)

No signatures of topological superconductivity...

Weyl SM junction: WTe₂



missing 1st step



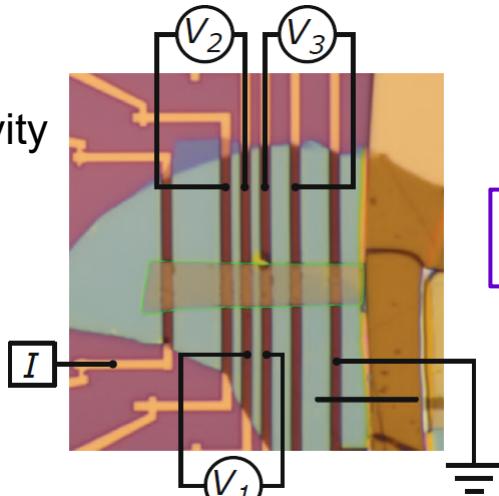
by Fabian Oppliger (master student 2020) & Artem Kononov et al.

WTe₂ “Fraunhofer effect”



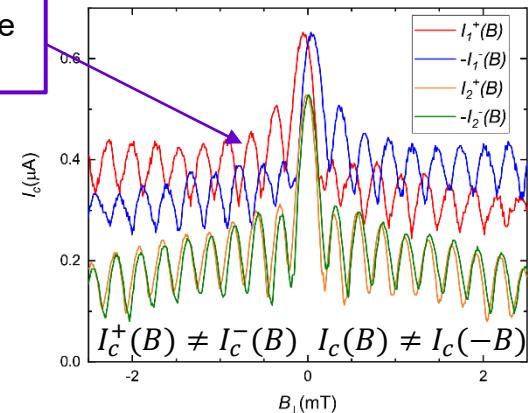
Pd induced superconductivity

$$I_c^+(B) \neq I_c^-(B)$$

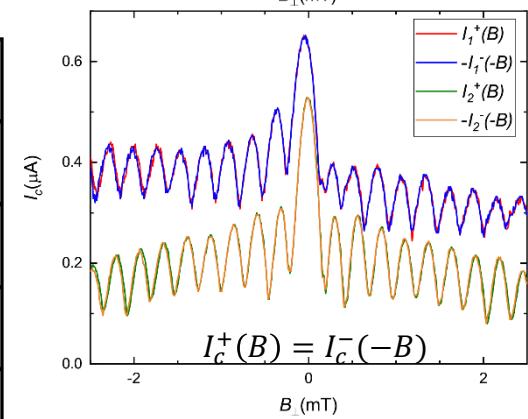
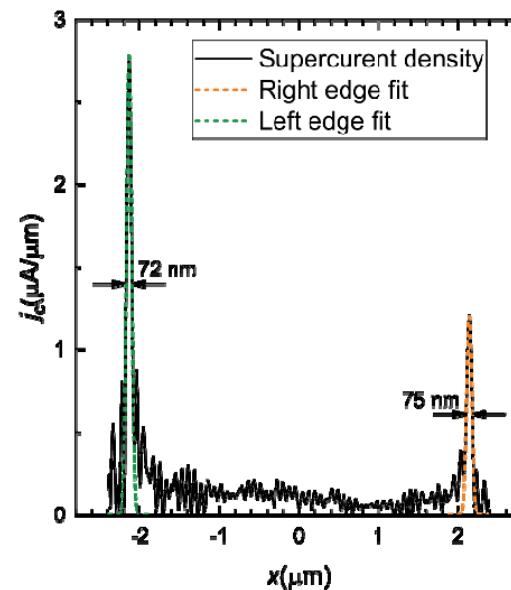
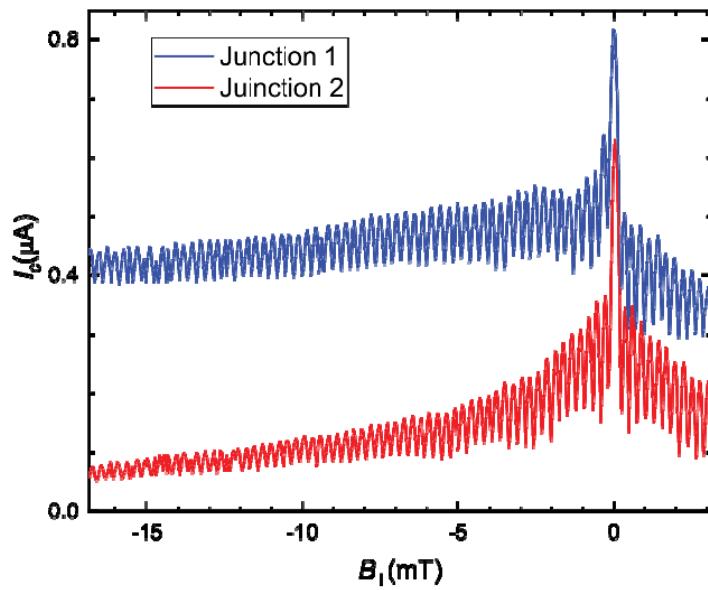


theory: Z. Wang et al., PRL 123, 186401 (2019)

Asymmetric Josephson effect
suggests non-sinusoidal CPR



Robust **SQUID-like oscillations** of $I_c(B_\perp)$ highlight presence of extremely narrow edge states

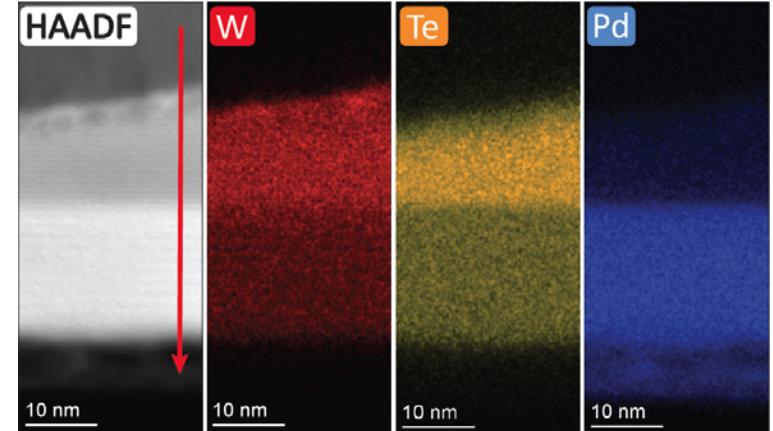
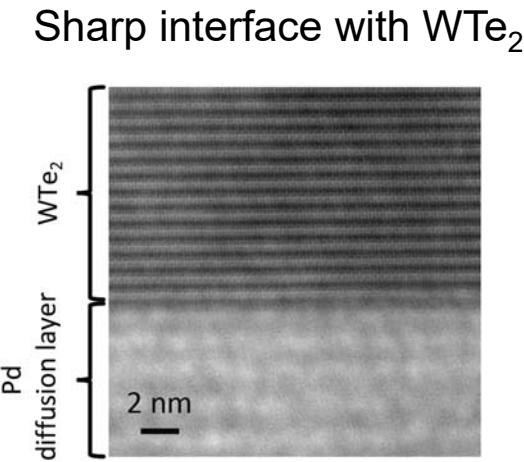
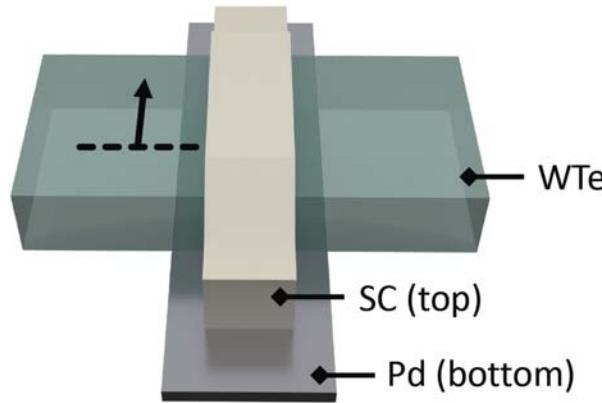


experiments:
Kononov et al., Nano Letters 20, 4228 (2020)
Y.-B. Choi et al., Nat. Mater. 19, 974 (2020)
C. Huang et al., Nat. Sc. Rev. 7, 1468 (2020)

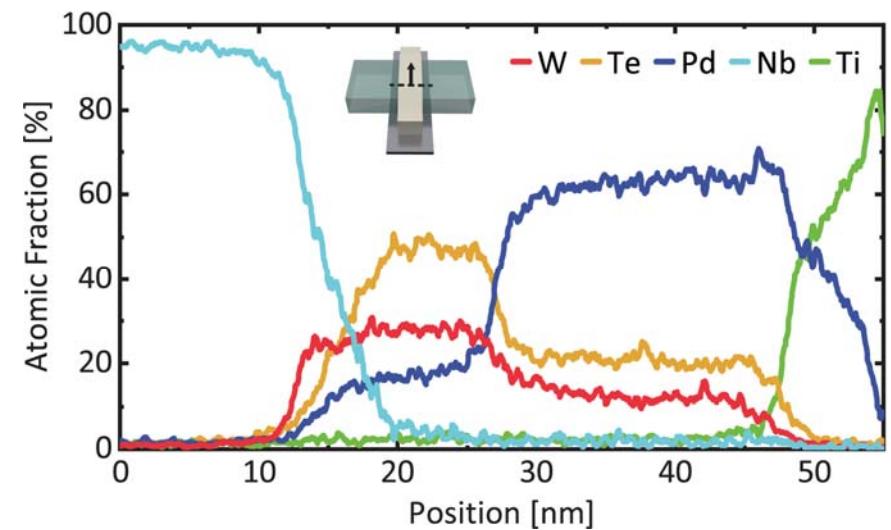
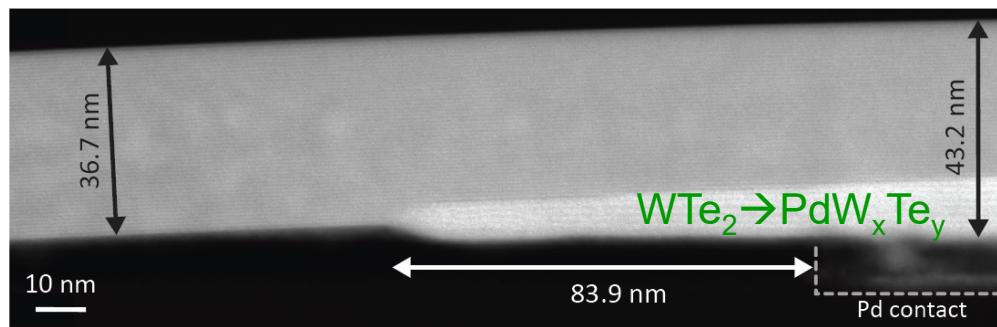
Origing of induced superconductivity



We investigated atomic structure with STEM/EDX

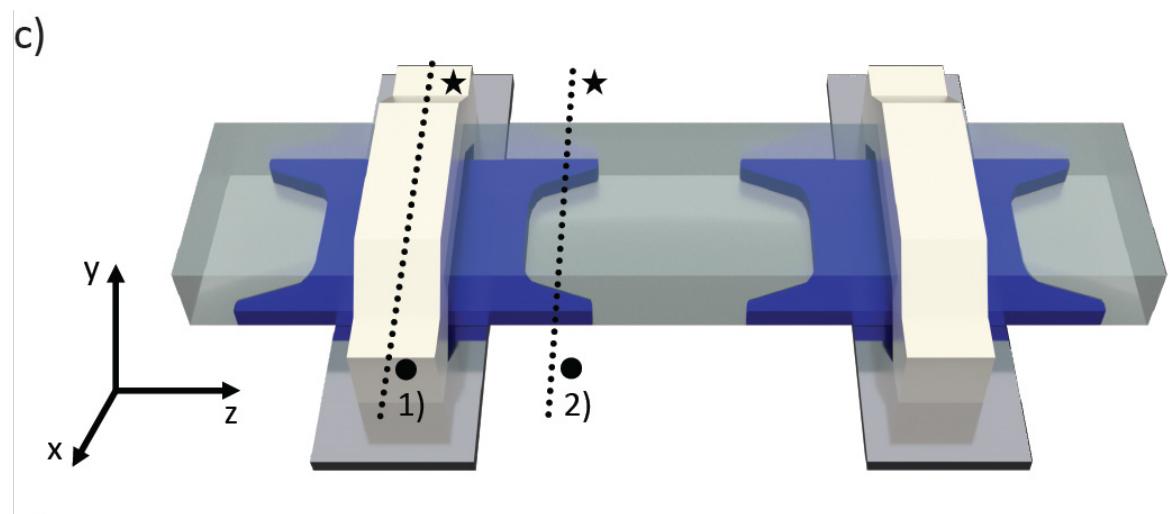
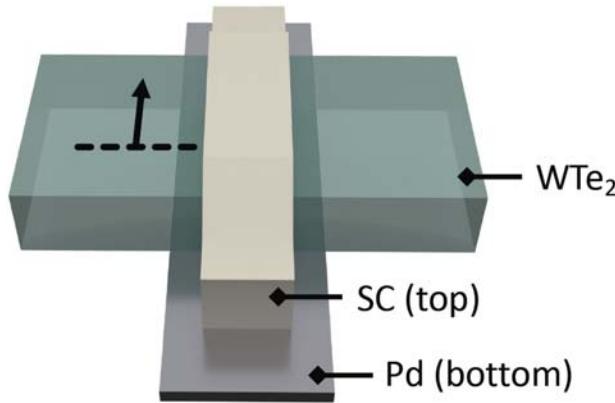


New crystal structure formed near Pd contacts

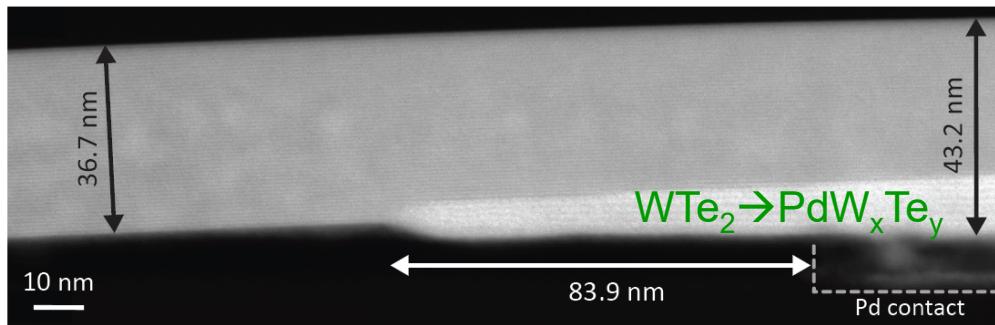


Origing of induced superconductivity

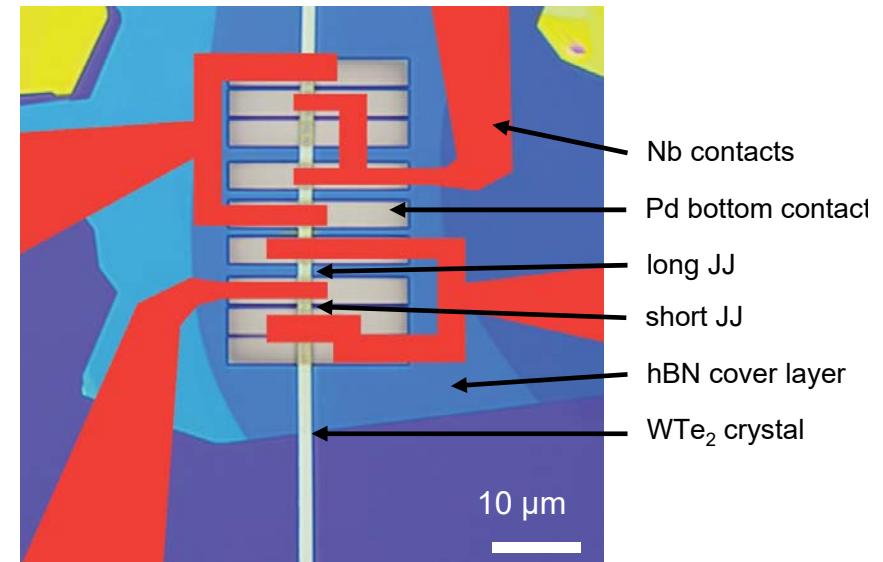
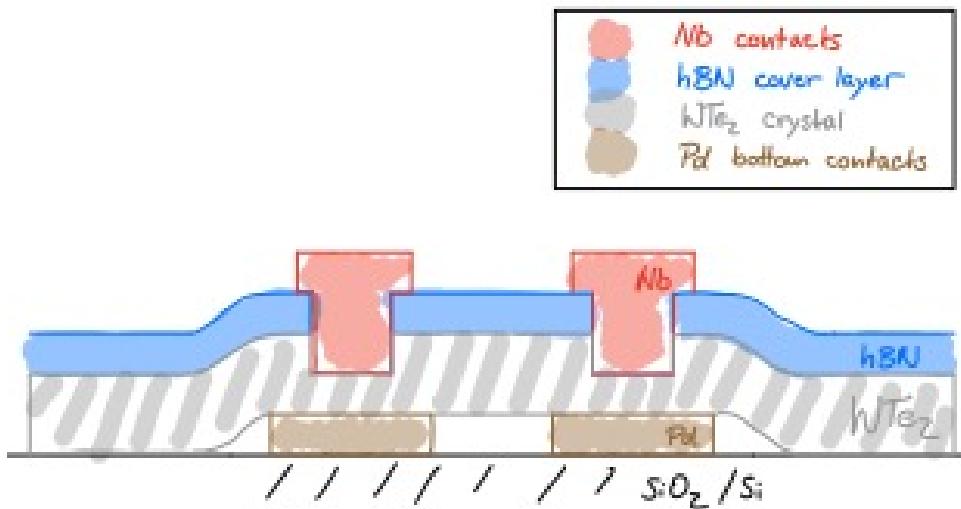
We investigated atomic
structure with
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New crystal structure formed near Pd contacts

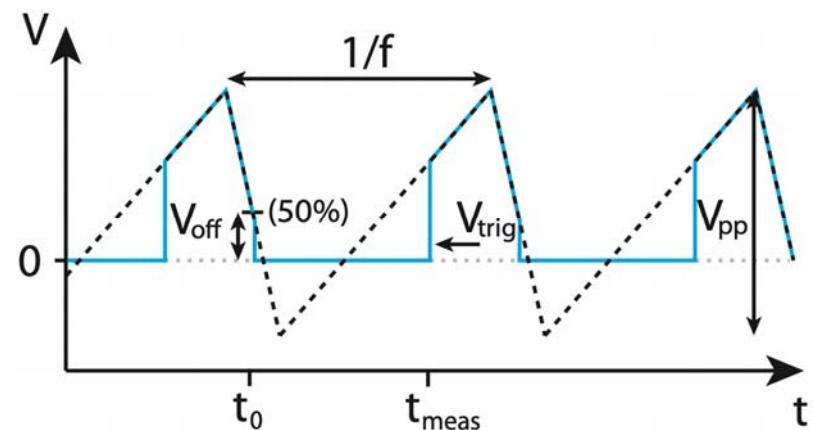


Asymmetric SQUID devices in WTe₂



Measurement of I_c with a counter

apply a current ramp and measure time when junction switches to the normal state, then repeat “over and over”



CPR of topological junctionn

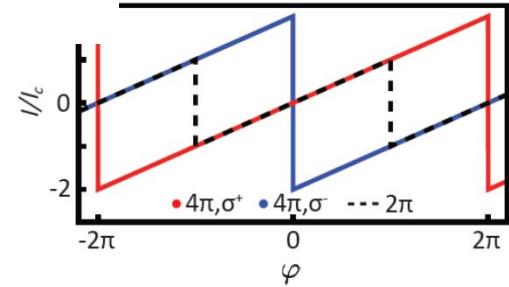
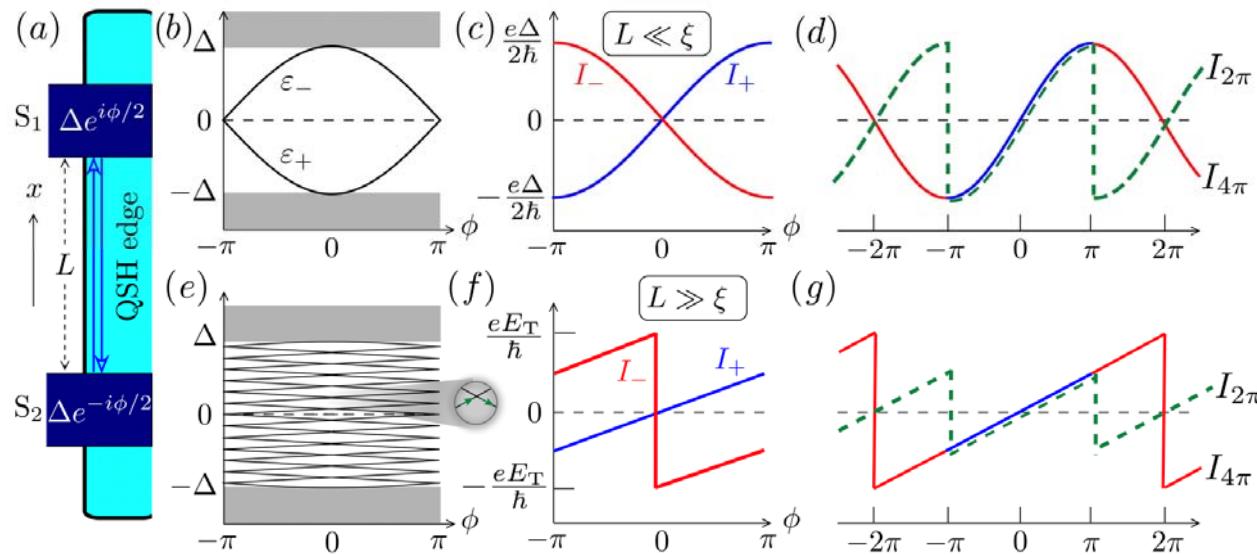


4π periodic SC in topological ballistic junction (with a helical edge)

PRL 110, 017003 (2013)

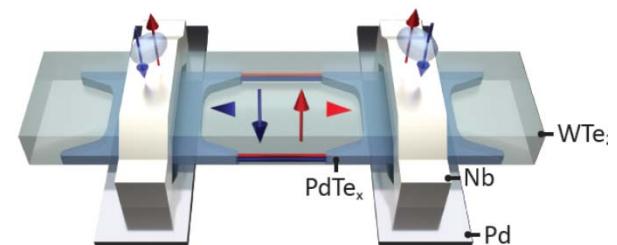
PHYSICAL REVIEW LETTERS

week ending
4 JANUARY 2013

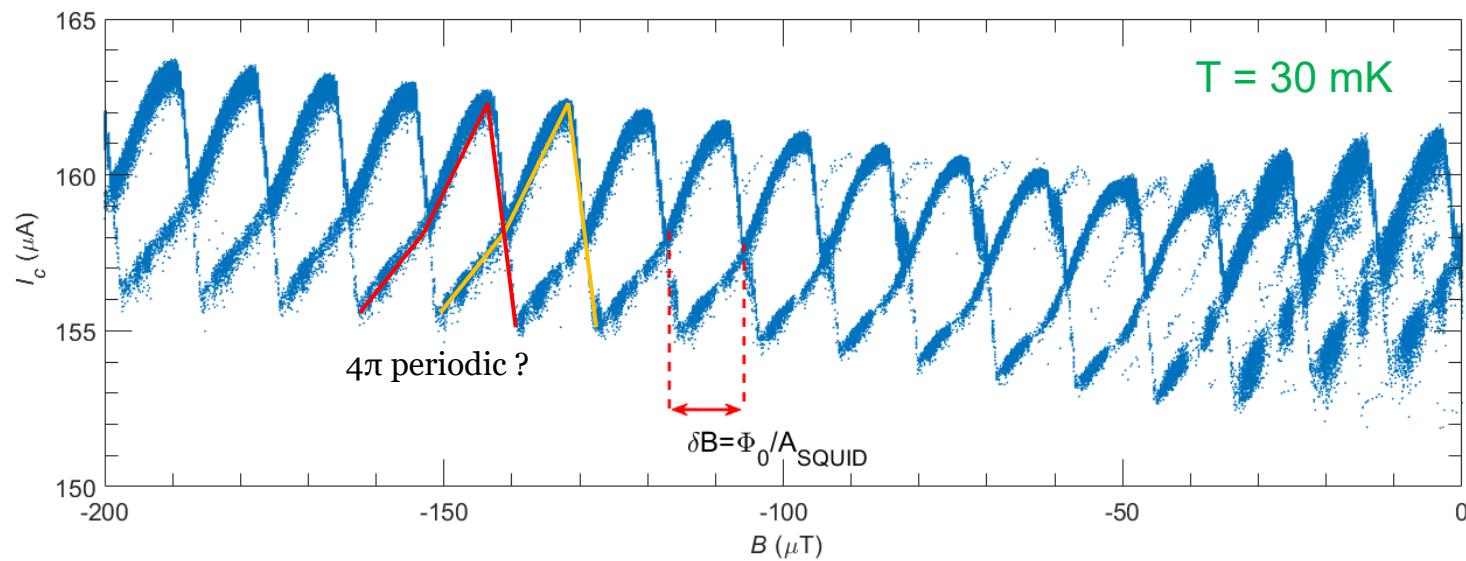
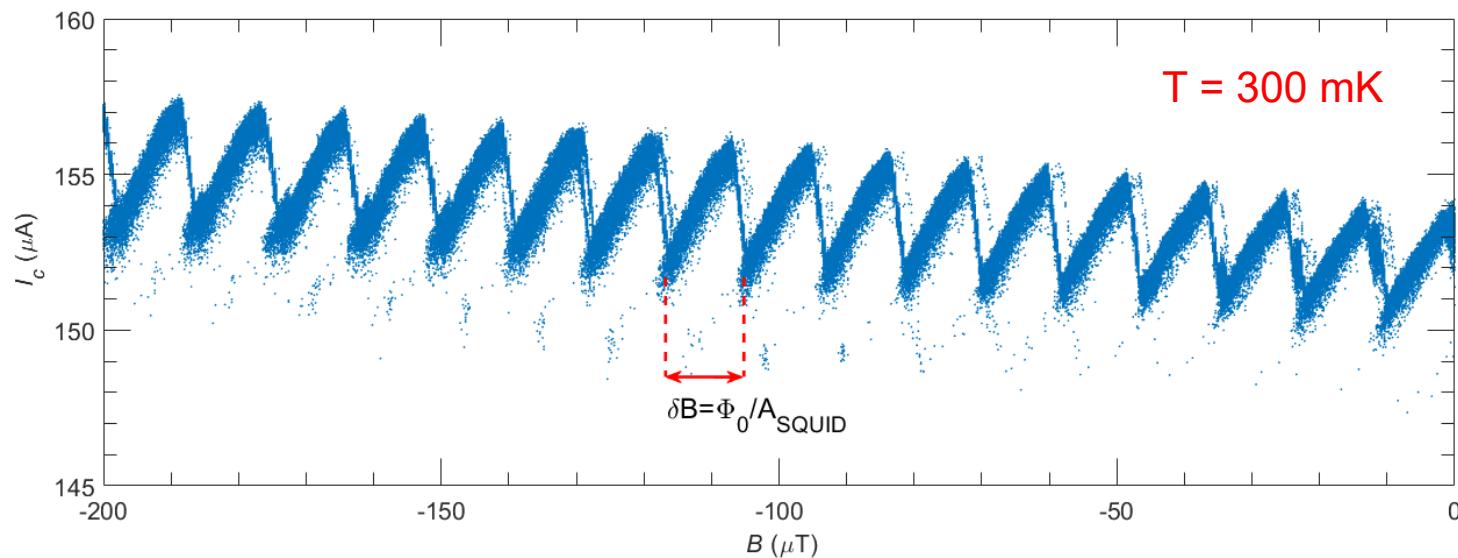


Beenakker et al., PRL 110 (2013)

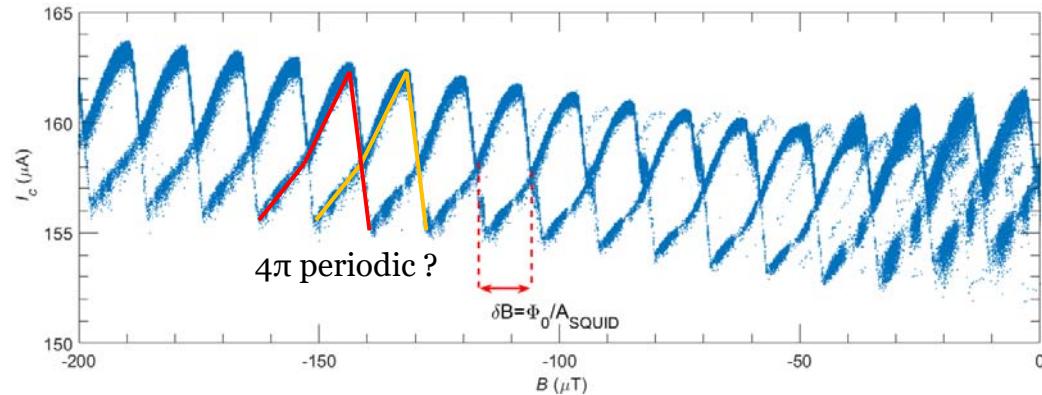
Fermion-Parity Anomaly of the Critical Supercurrent
in the Quantum Spin-Hall Effect



Interesting SQUID signals

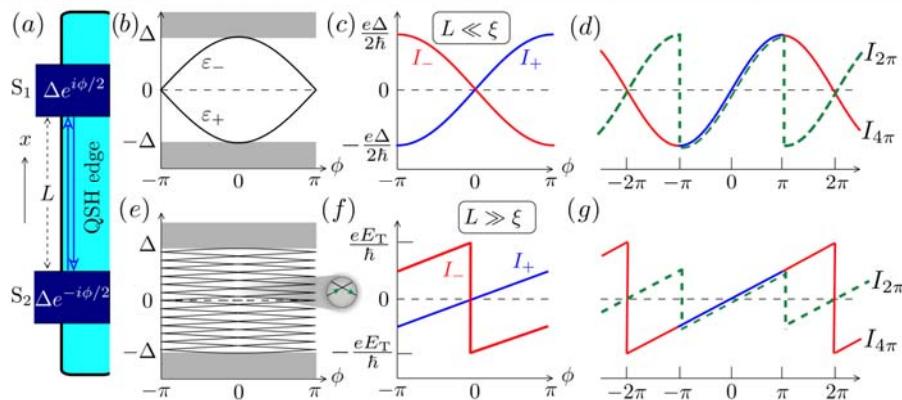


Interesting SQUID signals



4π periodic SC in topological ballistic junction

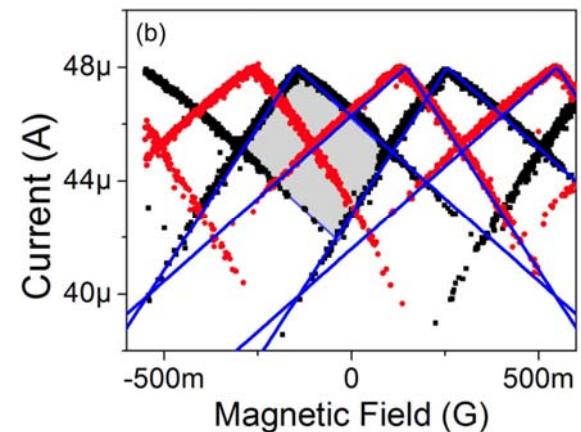
PRL 110, 017003 (2013) PHYSICAL REVIEW LETTERS week ending 4 JANUARY 2013



- 1) Doubled Amplitude
- 2) 4π periodicity

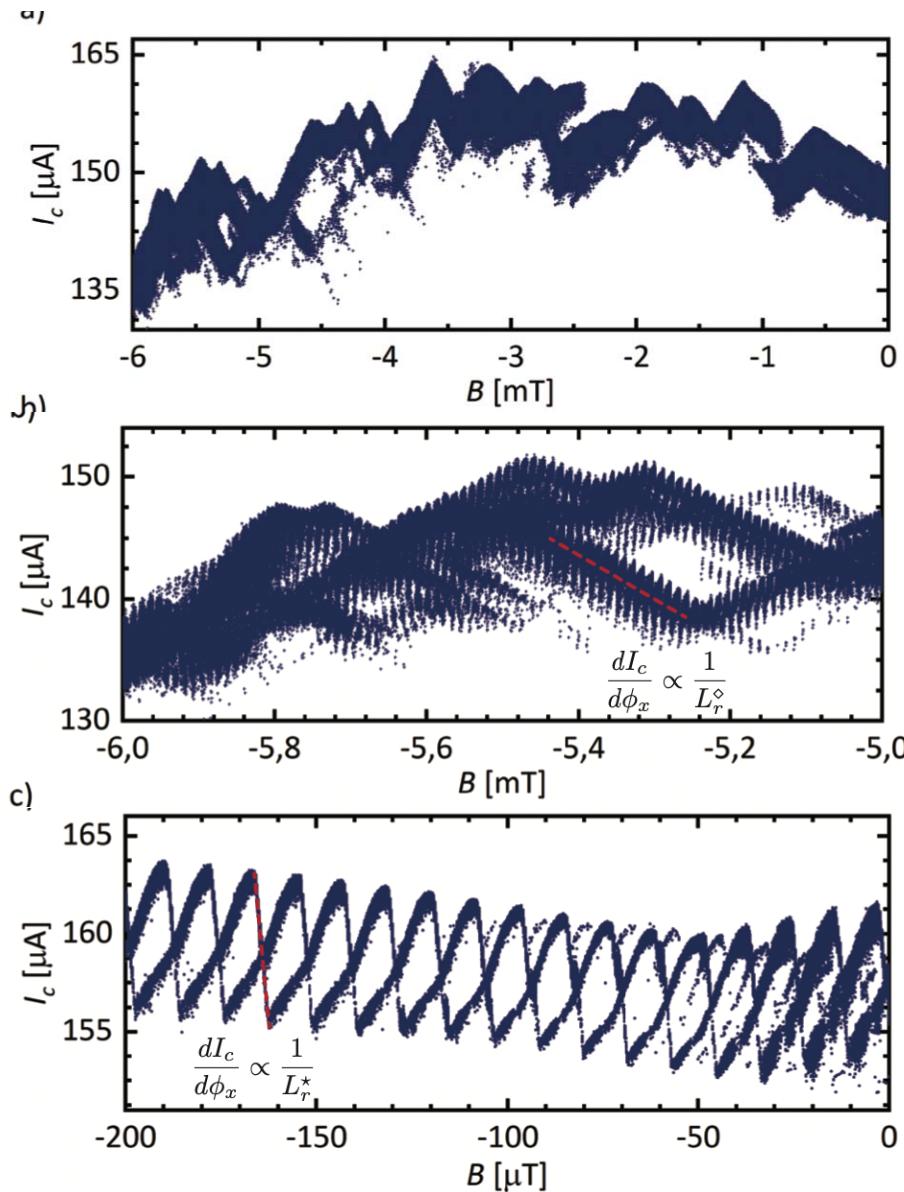
Beenakker et al., PRL 110 (2013)

Multivalued CPR, Little Parks diamonds



Murphy et al. , PRB 96, 2017

Much larger flux range: multivalued !



Increasing magnetic field resolution

Long range behavior attributed to reference junction

Multivalued I_c resembling **inductance** dominated SQUID

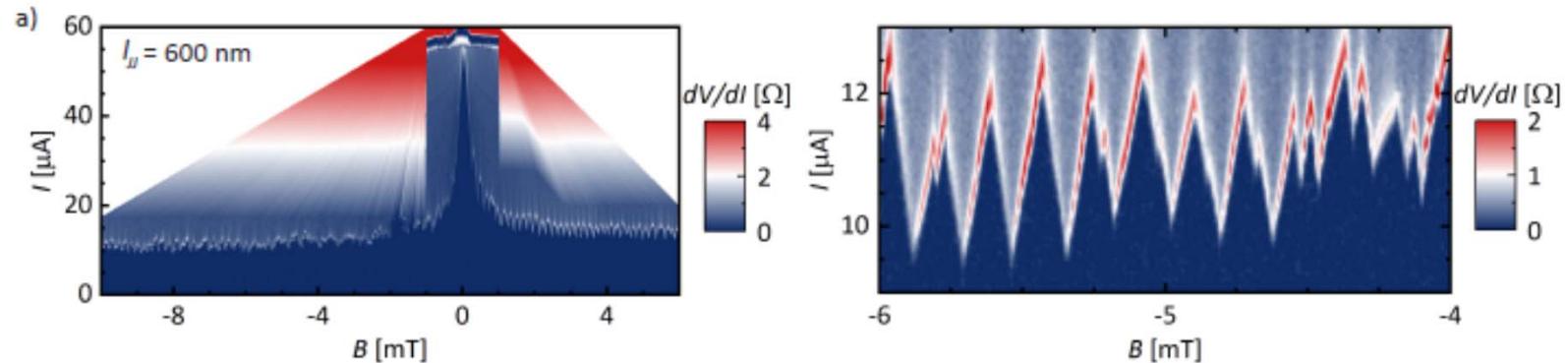
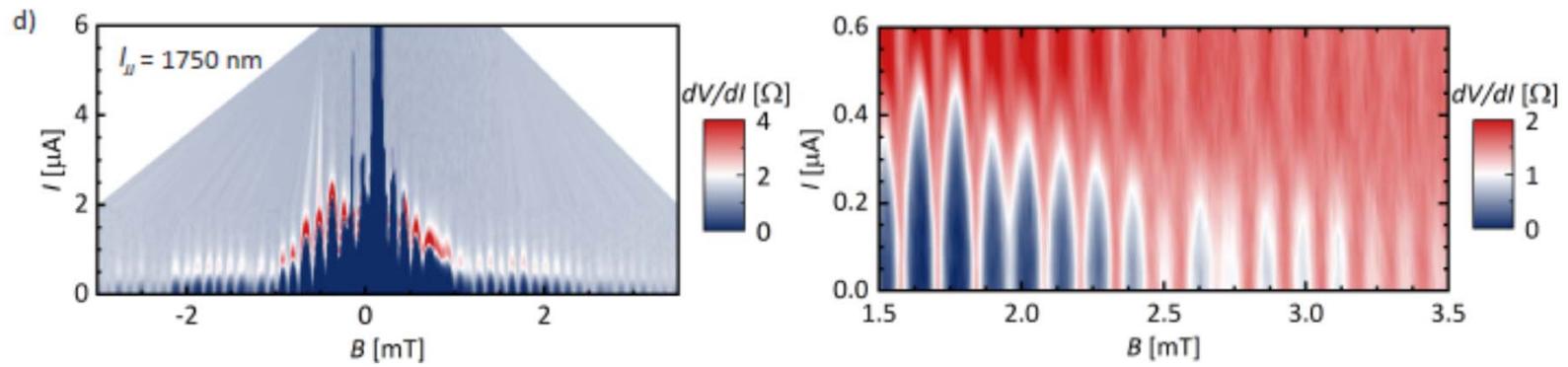
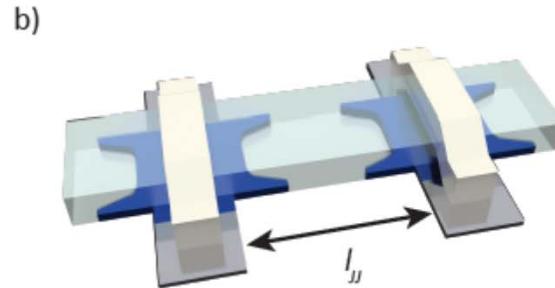
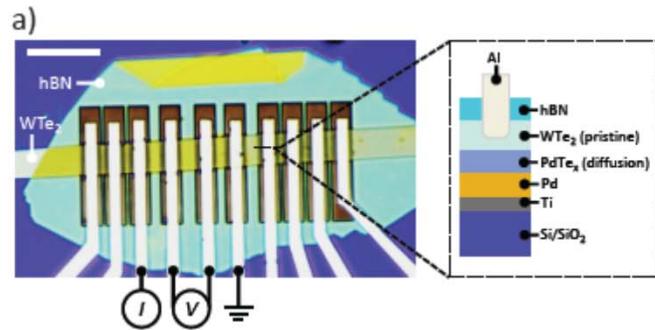
Matching periodicity of SQUID oscillations:

$$\delta B = 11.6 \mu\text{T}$$

$$\delta B = \Phi_0/A_o = 11.1 \mu\text{T}$$

- Murphy et al., PRB 96 (2017)
 Lefevre-Seguin et al., PRB 46 (1992)
 Friedrich et al., Appl. Phys. Lett. 104 (2014)
 Hazra et al., Appl. Phys. Lett. 16 (2021)
 Dausy et al., Phys. Rev. Appl. (2021)

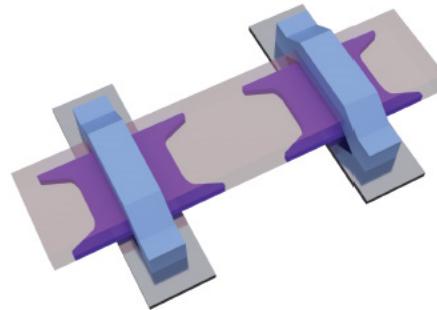
Single junction



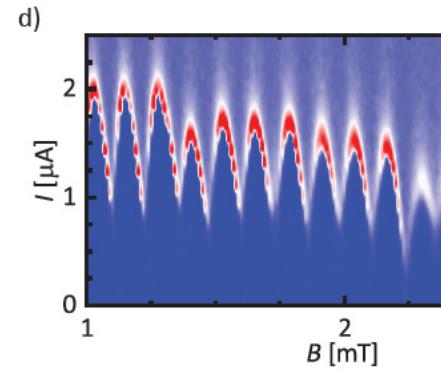
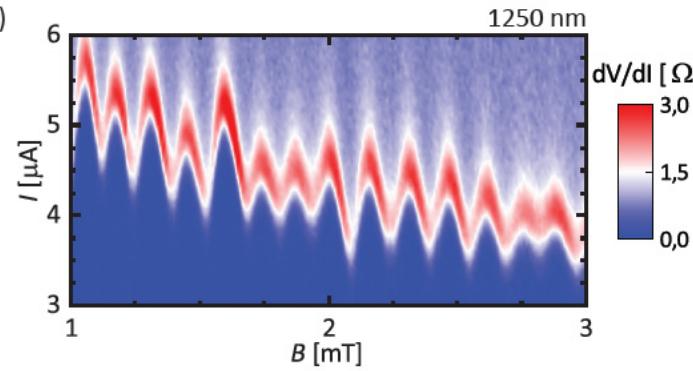
Single junction (also a sort of SQUID)



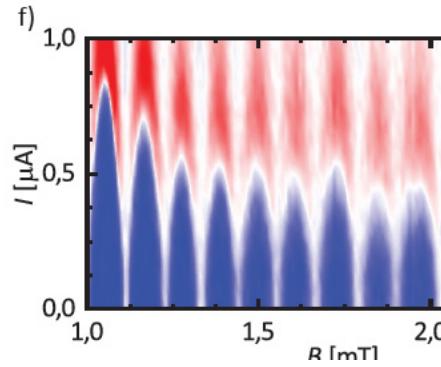
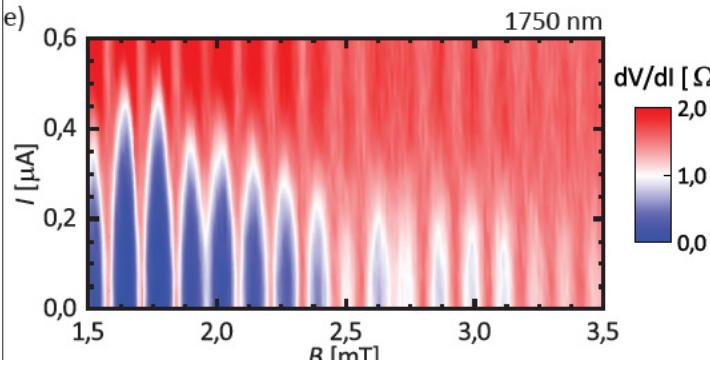
a)



c)

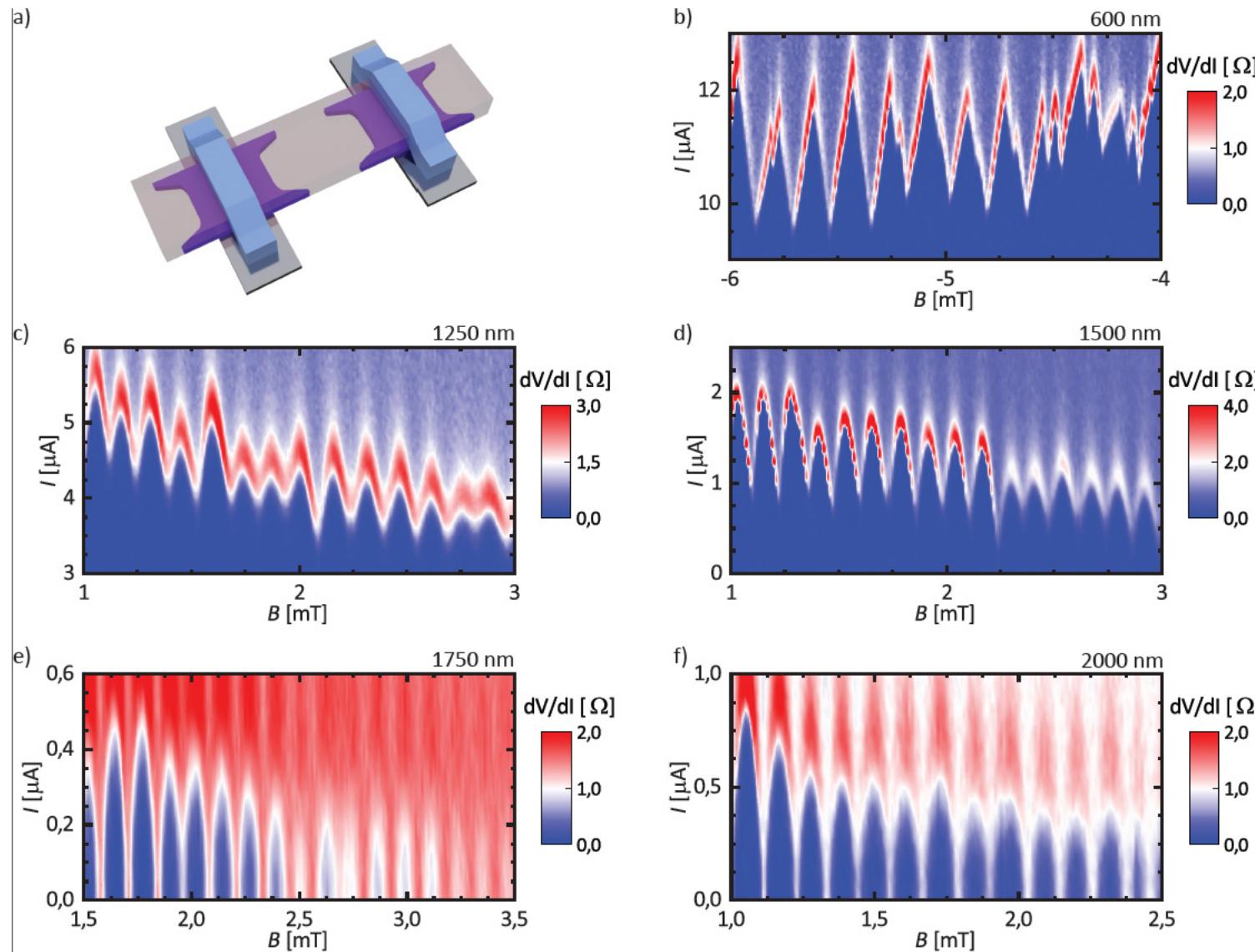


e)



there is still a large supercurrent even for separations as large as $2\mu\text{m}$

Single junction (also a sort of SQUID)



suggest a
S-S'-S weak link

there is still a
large super-
current even for
separations as
large as $2\mu m$

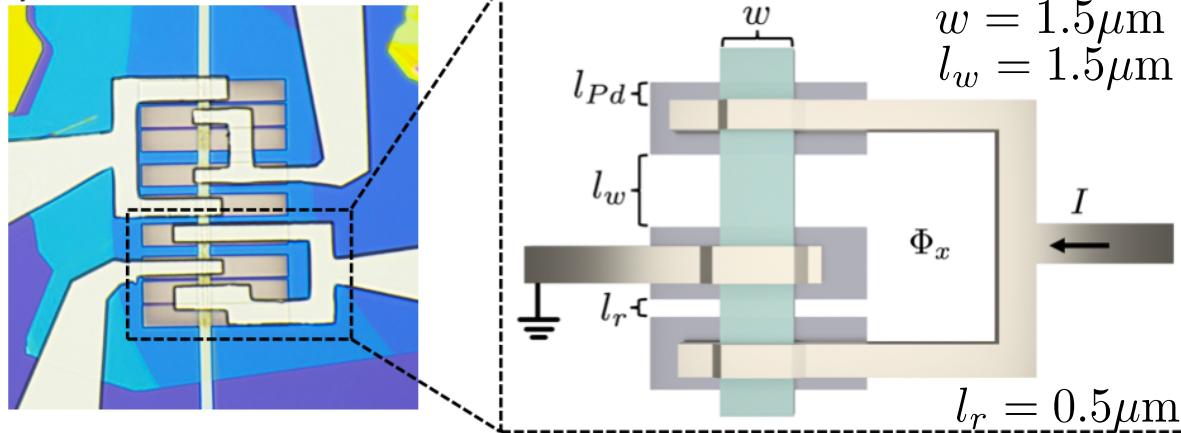
CPR without loop inductance



Suggests that there is **inductance** likely produced by the PdTe alloy.

For large loop inductance, relation between applied **flux and phase** over the weak junction is **no longer single-valued**. Moreover, phase at reference junctions is not fixed at $\pi/2$!

a)



assumptions:

strong asymmetry:

$$I_c^r \gg I_c^w \rightarrow \varphi_r = \varphi_r^{max}$$

no loop self inductance:

$$\varphi_w - \varphi_r = \phi_{tot} = 2\pi \frac{\Phi_{tot}}{\Phi_0}$$

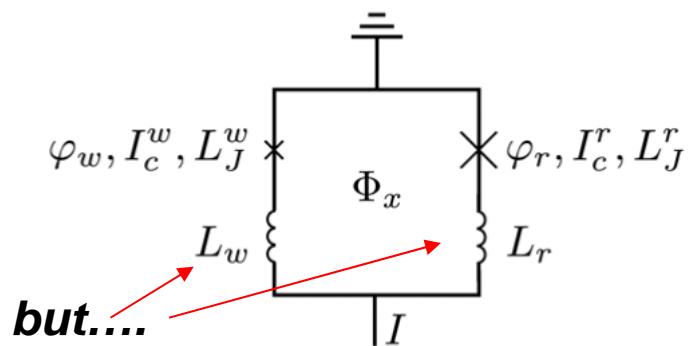
$$I_c = \underbrace{I_c^w f_w(\varphi_w)}_{I_w} + \underbrace{I_c^r f_r(\varphi_r)}_{I_r}$$

I_c^i critical current

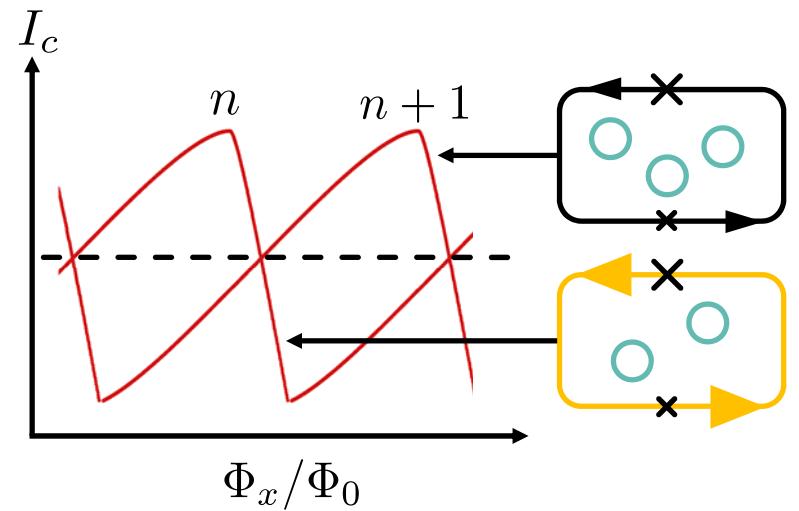
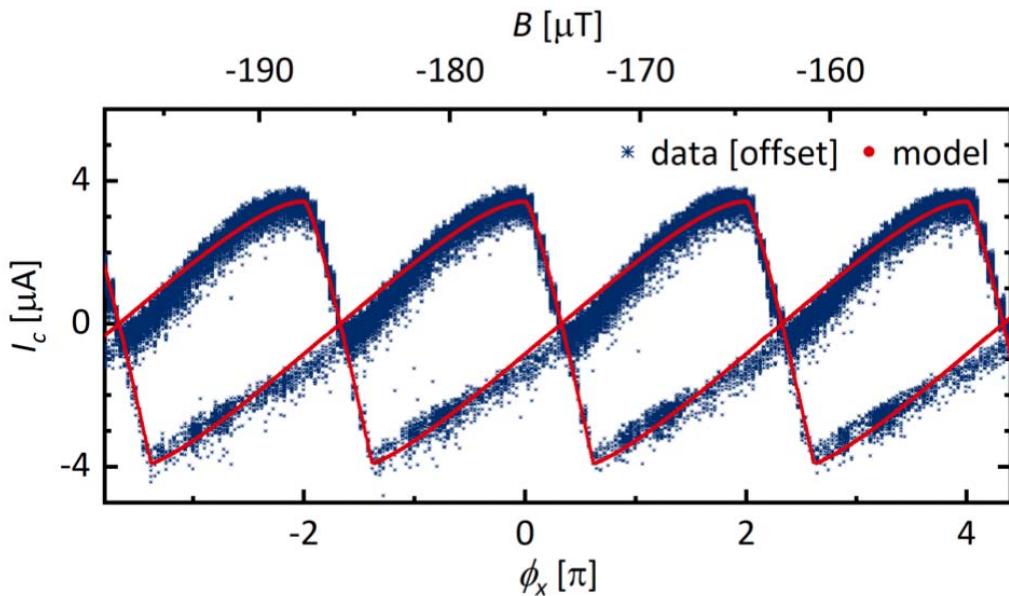
f_i normalized CPR

→ CPR of the weak junction:

$$I_c(\phi_{tot}) \sim I_c^w f_w(\varphi_r^{max} + \phi_{tot}) + I_c^r f_r(\varphi_r^{max})$$



CPR with loop inductance



Problem: inductance effects

→ **Approach:** CPR assumption

$$\phi_{tot} = \phi_x + \frac{2\pi(L_r I_r - L_w I_w)}{\Phi_0}$$

one can show that a crossing in $I_c(\phi_x)$ implies that the reference phase jumps

- 1) non-linearity in rising slope
- 2) Self-crossings

$$I_w(\varphi_w) = I_c^w \frac{\sin(\varphi_w)}{\sqrt{1 - \sin^2(\varphi_w/2)}}$$

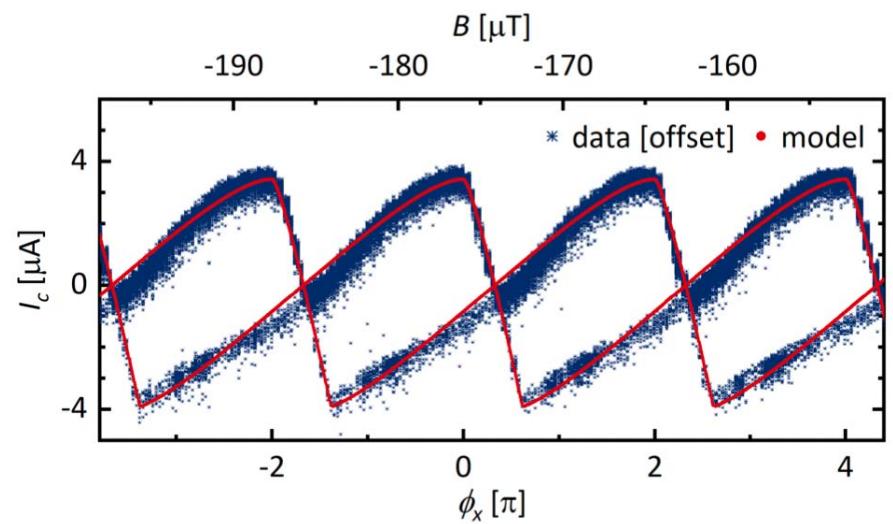
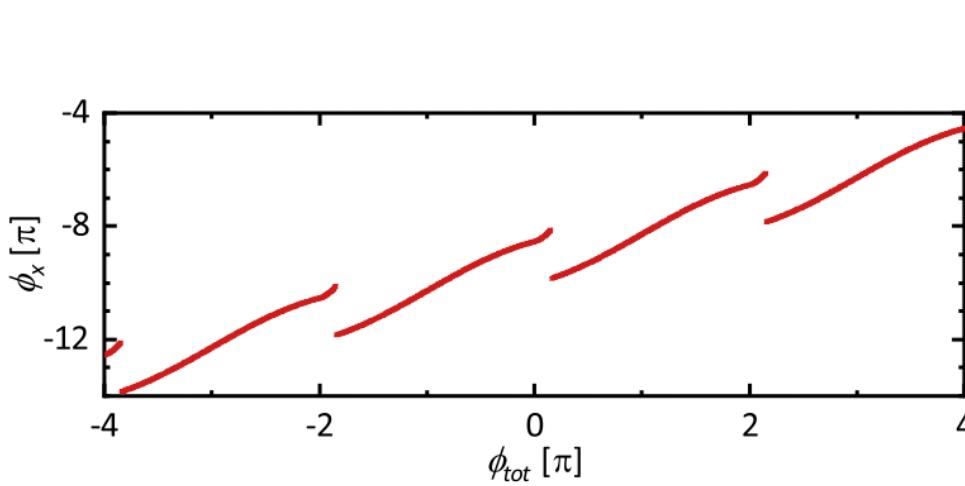
CPR with loop inductance



- 1) Maximize $I_c(\phi_{tot}) = I_r(\varphi_r(\phi_{tot})) + I_w(\varphi_r(\phi_{tot}) + \phi_{tot})$ with respect to $\varphi_r(\phi_{tot})$
- 2) Extract the inductance effects $\phi_x = \phi_{tot} - 2\pi(L_r I_r - L_w I_w)/\Phi_0$
- 3) Plot $I_c(\phi_x)$

From the fit:

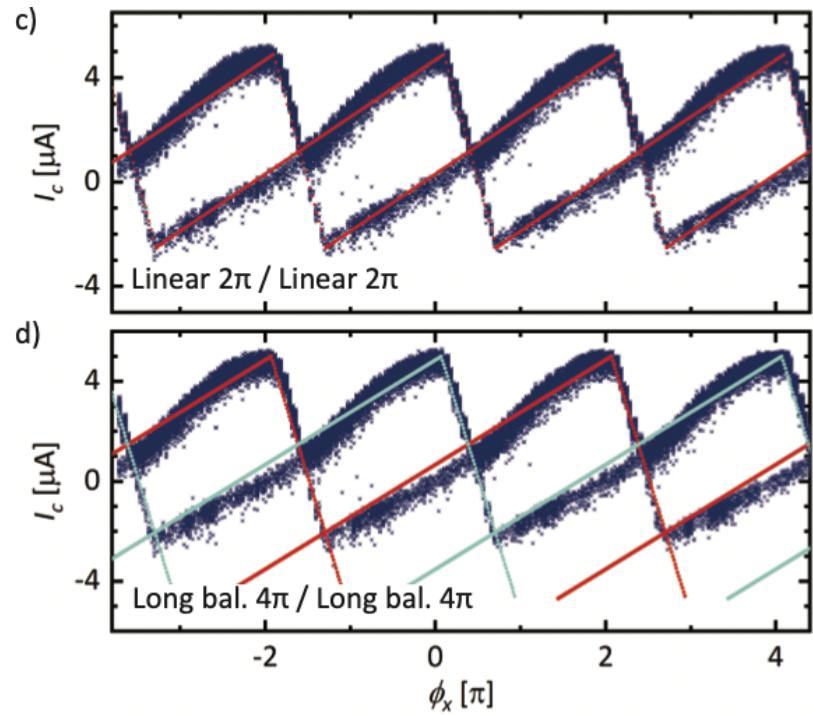
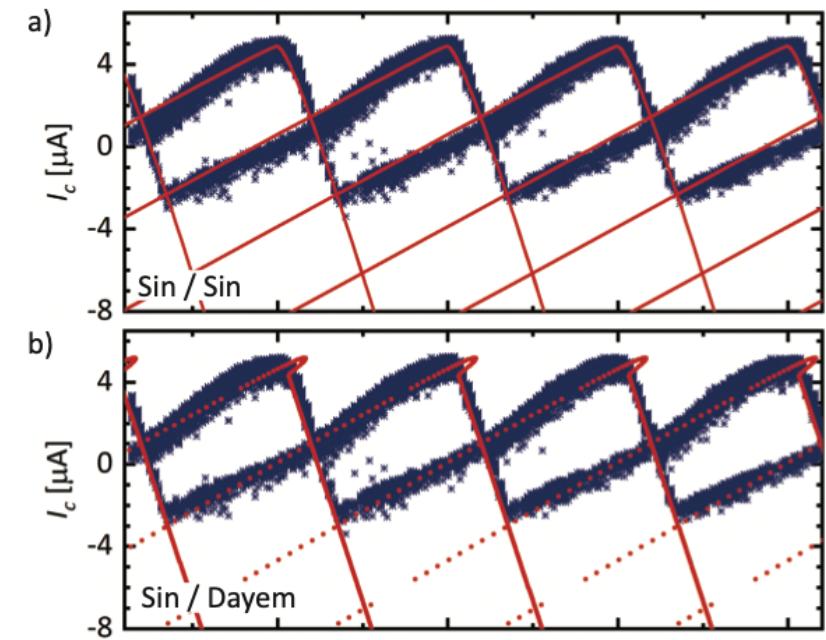
$$\left. \begin{array}{l} L_r = 60 \text{ pH} \\ L_w = 220 \text{ pH} \end{array} \right\} \text{Exceeds } L_{geo} \approx 27 \text{ pH and } L_{kin} \approx 5.5 \text{ pH}$$



Shatz et al., PloS one 9 (2014)
Annunziata et al., Nanotechnology 21 (2010)

Inductance of the reference junction

Comparison of fit models

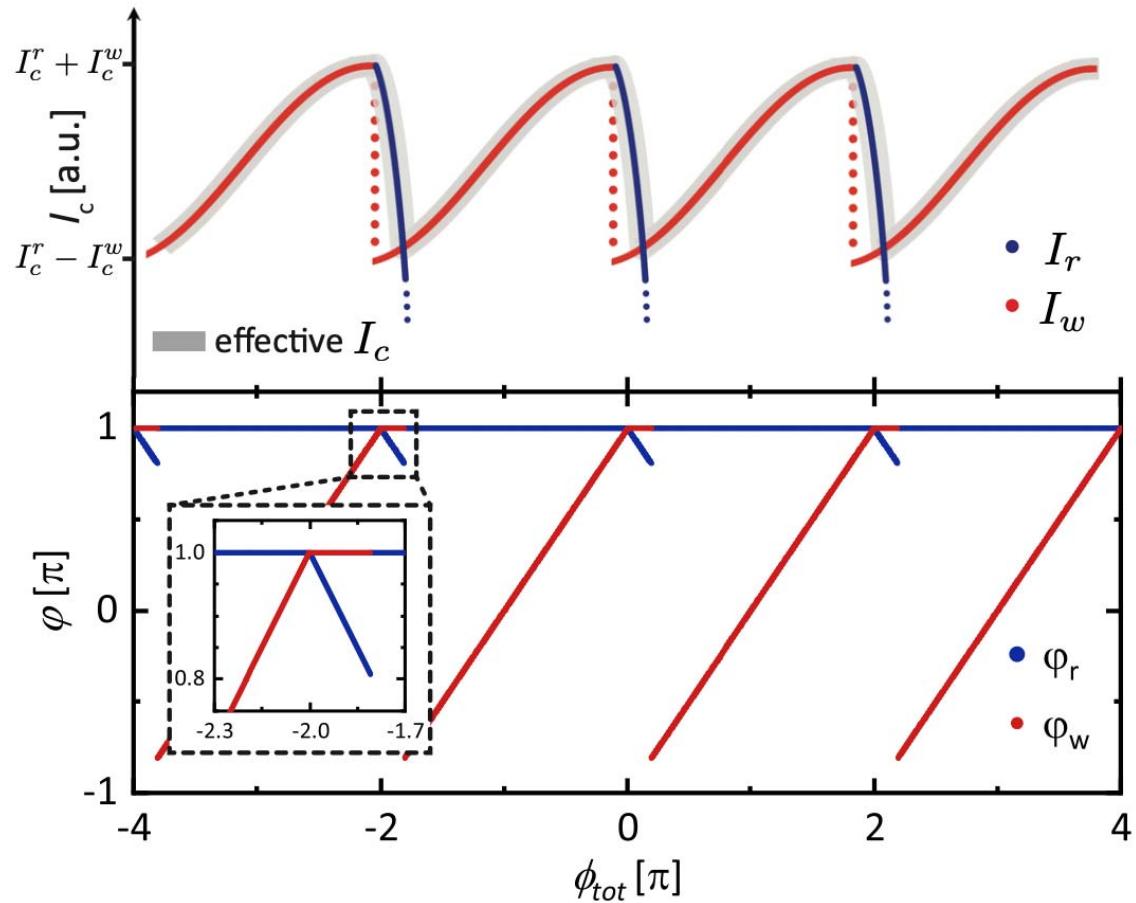


f_w f_r	$\text{Sin}(\varphi)$ $\text{Sin}(\varphi)$	$\text{Sin}(\varphi)$ Dayem	$\text{Linear } 2\pi$ $\text{Linear } 2\pi$	$\text{Long bal. } 4\pi$ $\text{Long bal. } 4\pi$
I_c^w [μA]	60	10	3.8	5
L_w [pH]	450	400	200	80
I_c^r [μA]	104	154	160	160
L_r [pH]	270	-	80	80

graphical illustration



Visual approach to maximize I_c



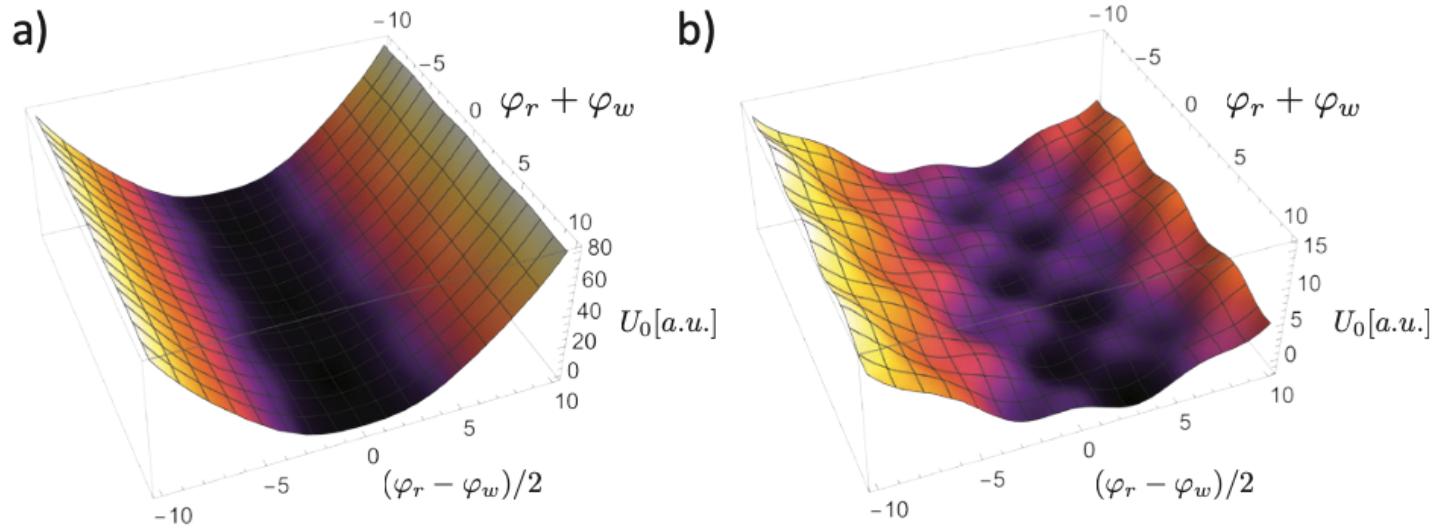
Goal: maximize I_c

$$\varphi_r - \varphi_w = \phi_{tot}$$

Both junction phases evolve in total flux

I_c is composed out of I_w and I_r

Another approach to multivalued SQUID



$$U(x, y) = U_0 \left[-\frac{I}{2I_0}x - \cos(x)\cos(y) - \alpha \sin(x)\sin(y) - \eta \frac{I}{2I_0}y + \beta \left(y - \frac{1}{2}\phi_x\right)^2 \right]$$

$$x = \varphi_r + \varphi_w$$

$$I = \frac{V_{bias}}{R_{bias}}$$

$$I_c^r = I_0(1 + \alpha)$$

$$y = (\varphi_r - \varphi_w)/2$$

$$I_c^w = I_0(1 - \alpha)$$

$$\beta = \frac{\Phi_0}{2\pi L I_0}$$

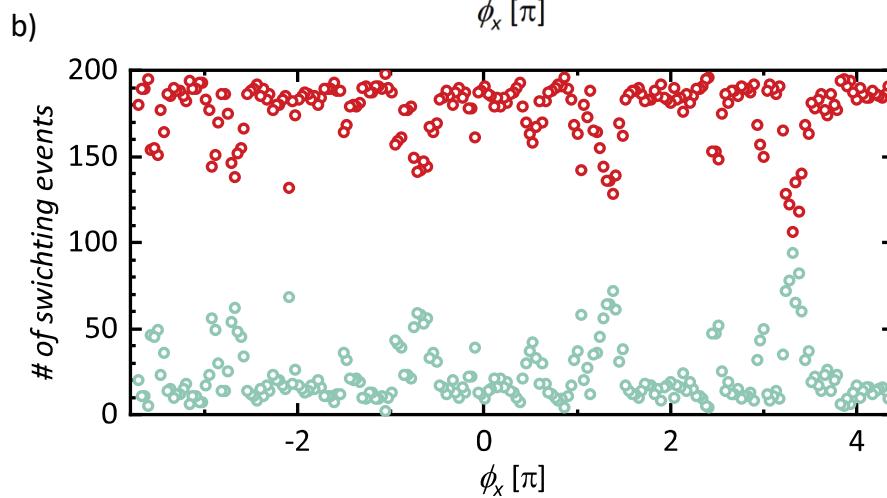
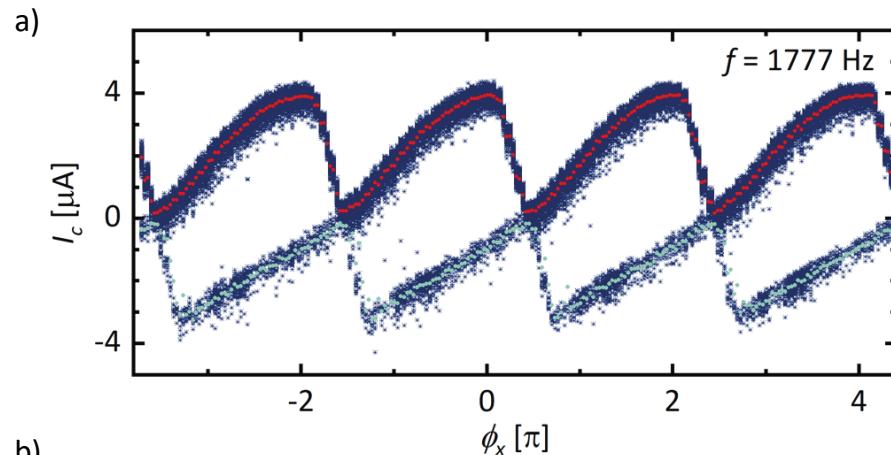
$$\alpha = \frac{a - 1}{a + 1}$$

$$a = I_c^r / I_c^w$$

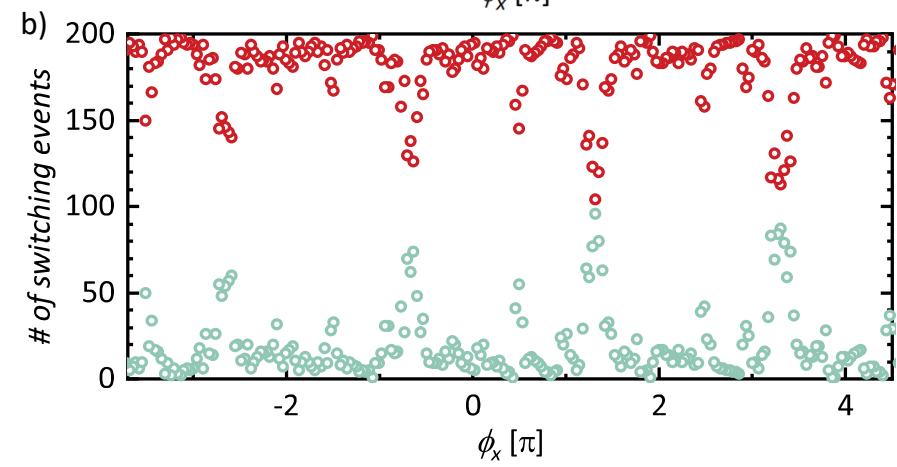
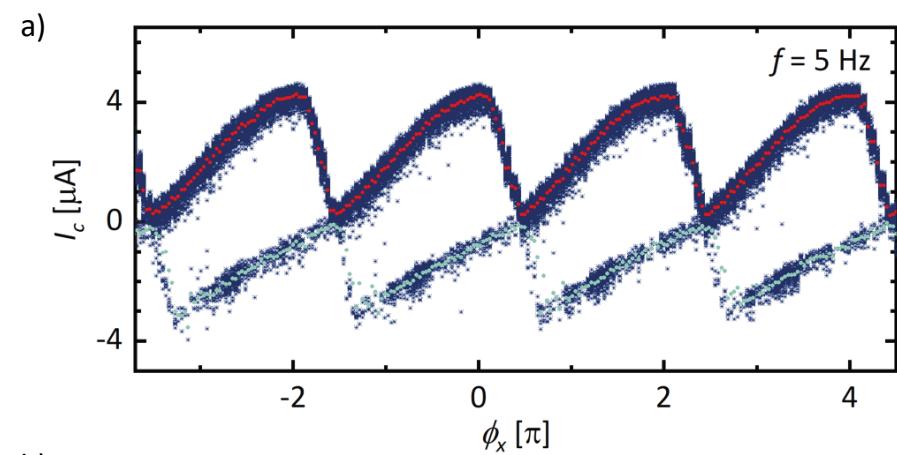
Lefevre-Seguin et al., PRB 46 (1992)

Another approach to multivalued SQUID

Occupation of vorticity states



$$r \sim 0.128$$



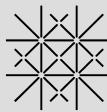
$$r \sim 0.107$$



Conclusions



- **no sign** of 4π -periodic current-phase relation
- **no sign** of the fractional Josephson effect (AC Josephson current mediated by single electrons, not Cooper pairs)
- **supercurrent over 1.5μm** is still quite impressive



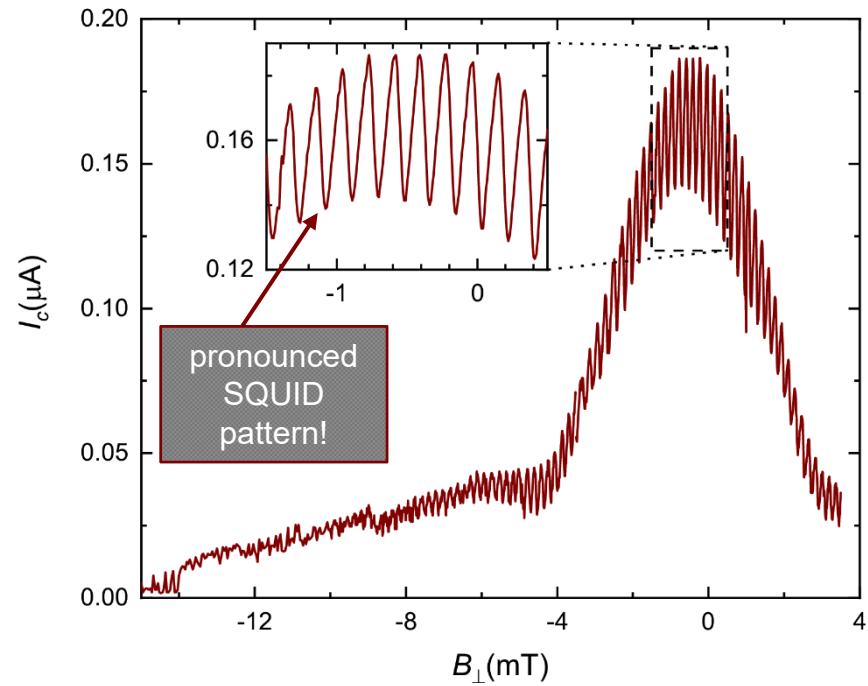
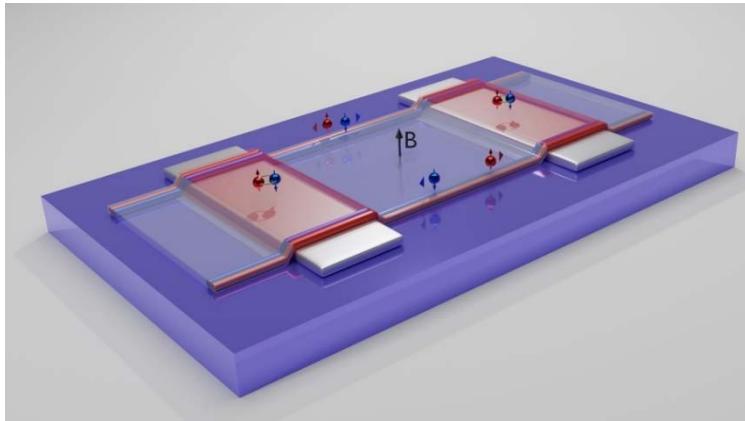
University
of Basel

Thank you for your attention!

Christian Schönenberger, 14th Oct. 2022



WTe₂ D. Mandrus et al. Univ. of Tennessee



A. Kononov et al. One-dimensional edge transport in few-layer WTe₂, Nano Letters 20, 4228–4233 (2020)

M. Endres et al. Transparent Josephson Junctions in Higher-Order Topological Insulator WTe₂ via Pd Diffusion, Phys. Rev. Mat. 6, L081201 (2022)(2022)

End of Lecture II

(maybe add results from Orsay –
Bouchiat's group)